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reinforced concrete member
subject to axial load and flexure

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THE DETERMINATION OF STRESSES IN
A REINFORCED CONCRETE MEMBER
SUBJECT TO AXIAL LOAD AND FLEXURE

BY

SIMON H INGBERG

C. E. University of Minnesota, 1909

THESIS

Submitted in Partial Fulfillment of the Requirements for the

Degree of

MASTER OF SCIENCE

IN THEORETICAL AND APPLIED MECHANICS

IN

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OF THE

UNIVERSITY OF ILLINOIS

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I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

SIMON H INGBERG

ENTITLED The Determination of Stresses in a Reinforced
Concrete Member Subject to Axial Load and Flexure

BE ACCEPTED AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE
DEGREE OF Master of Science

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Combined Thrust and Bending

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INTRODUCTION

The use of concrete and reinforced concrete in arch bridges has necessitated the deduction of formulae for obtaining the value of stresses in steel and concrete when the section considered is subject to combined thrust and flexure. The resulting equations are quite complex and must be graphically represented by diagrams to admit of ready use in design.

The experimental study of the behavior of reinforced concrete under the above named conditions had, to the writer's knowledge, not been made until the present series of tests were undertaken. Such a study appeared desirable from the standpoint of design on account of the complexity of conditions involved. The plain concrete specimens also afforded opportunity for ascertaining the qualities of concrete in flexural tension and the reinforced ones the same with regard to compression, the amount of the latter force present being generally greater than when the section is subject to simple flexure.

The experiments were undertaken as a part of the investigational work of the Engineering Experiment Station of the University of Illinois, and was under the immediate charge of Prof. A. N. Talbot, to whom, as well as to other members of the staff, acknowledgments are due for valuable suggestions and aid.

Before proceeding to describe the experiments, the following theoretical consideration will be given, which is identical with that presented in the writer's thesis, University of Minnesota, 1909.

The Theoretical Determination of Stresses in a Reinforced Concrete Section Subject to Normal Load and Flexure.

When a section is subjected to thrust that acts outside of the center of gravity of the transformed composite section of concrete and steel, a bending moment is produced that modifies the amount and distribution of the stresses due to normal force alone. If R be the resultant of all external forces acting on the section and N its component perpendicular to the section, the resulting bending moment M will then be equal to Ne' , where e' is the distance between the point of application of N and the center of gravity of the transformed section.

The center of gravity of a transformed reinforced concrete section being dependent on the extent to which the concrete is considered intact as well as upon the amount and position of reinforcement, it is simpler to take moments with reference to the geometric center of the section. This involves no approximation, since N remains the same in position and magnitude. In the graphical work N , and its distance e from the center of figure, can be scaled from the diagram employed. When M and N are obtained analytically or from summations, the axis with reference to which M has been found must be stated in order that it may be reduced to geometric axis. Thus, for a moment M' and a normal force N found with reference to an axis through ^{the} center of gravity situated a distance x from the center of figure toward ^{the} compressive side of the section $e' = \frac{M'}{N}$ and $e = e' + x$ is the value considered in the following discussion. As a rule, in arch

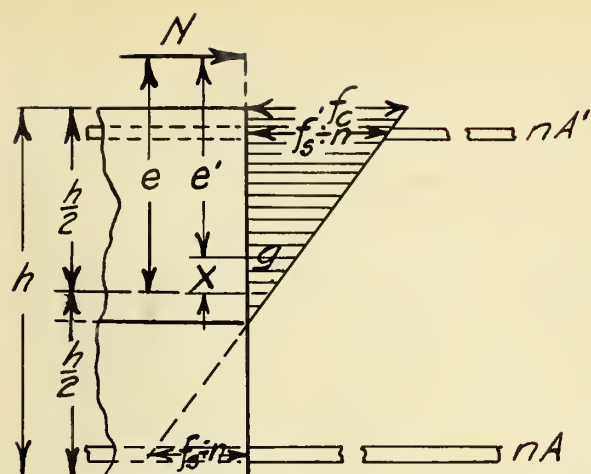


Fig. 1

analysis, moments are obtained analytically with reference to geometric axes so no reduction is necessary. Fig. 1 shows the stresses in a transformed section with no tensile resistance in concrete, and steel areas A and A' each respectively regarded equal to nA and nA' in determin-

ing the center of gravity, n being ^{the} ratio

$$\frac{\text{Modulus of Elasticity of Steel}}{\text{Modulus of Elasticity of Concrete}} = \frac{E_s}{E_c}$$

The Determination of stress in steel and concrete falls under three cases, the difference in method of treatment being due to the difference in intensity of tensile stress which different combinations of moments, loads and sections may produce, and to different assumptions as to the distribution of stress across the section.

Case I applies to sections where ^{the} stress is either wholly compressive or the tensile stress if present is not beyond the allowable limit for concrete.

Case II is used when the tensile unit stress exceeds the allowable limit, the concrete on the tension side of the neutral axis being considered to offer no resisting moment. Case I and II assume the stress to be distributed throughout the section according to the linear law, the intensity at any point being proportional to its distance from the neutral axis.



Case III, used for determining stresses at ultimate failure, assumes a parabolic distribution of the compressive stress in the concrete, the stress in the outer fiber becoming one-half of what it would be with linear variation for the same deformation, whereas the steel stress, which must be below the elastic limit in order that initial failure due to compression may occur, follows the linear law. The above assumption is justified by the experimental fact that the stress-strain curve for concrete in compression closely resembles a parabola, the modulus of elasticity varying from an initial value of 2 000 000 or more lb. per sq. in. to one-half of this value at maximum load. For this case, as for the two preceding, the deformed section is assumed plane so that unit stress and strain are proportional for all cases where the linear law governs.

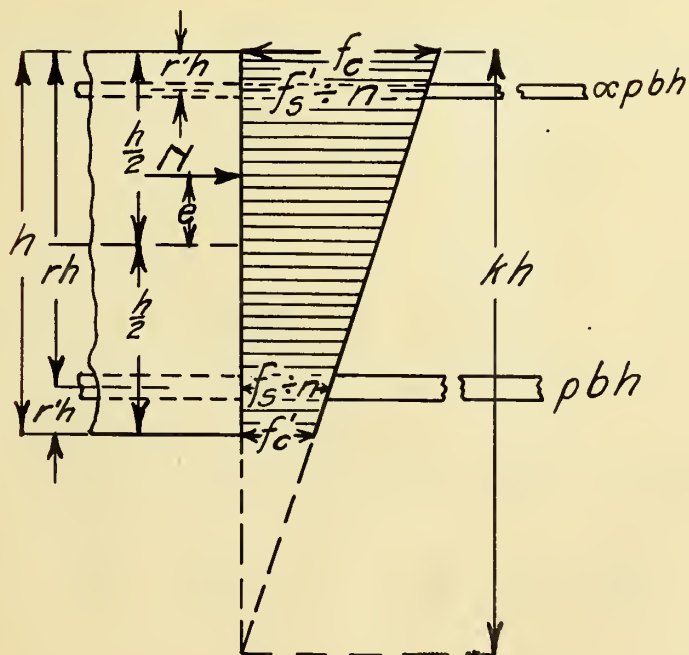


Fig. 2

crete stress in outer fiber remote from N. f_c' may be com-

Case I.

Stress over Full Section.

Let N be the resultant of all normal forces acting on the section and e its distance from the center of the section. Let f_c be the unit compressive stress in outer fiber on side nearest to N and f_c'

the corresponding con-



pression or an allowable tension. Fig. 2 shows the stress wholly compressive. Let h be the height and b the breadth of the section, and consider the reinforcements A' and A to be situated distances $r'h$ and rh respectively from outer fiber nearest N . For convenience $A = pbh$, where p = steel ratio and $A' = p'bh = \alpha pbh$. Also $r'h = h - rh$, whence $r' = (1 - r)$. f_s and $f_{s'}$ are the unit stresses in steel at A and A' , respectively, each being equal to the concrete stress at the point times $n = \frac{E_s}{E_c}$. Then from the assumption of linear variation of stress,

$$\frac{f_{c'}}{f_c} = \frac{kh - h}{kh} = 1 - \frac{1}{k} \dots \dots \dots (1)$$

$$\frac{f_{s'}}{nf_c} = \frac{kh - r'h}{kh} = \frac{k + r - 1}{k} \dots \dots \dots (2)$$

$$\frac{f_s}{nf_c} = \frac{kh - rh}{kh} = \frac{k - r}{k} \dots \dots \dots (3)$$

$$\frac{1}{2} (f_c + f_{c'}) bh + \alpha pbh f_{s'} + pbh f_s - N = 0, \quad (4)$$

since normal components equal zero; also, taking moments about N

$$\begin{aligned} & \frac{1}{2} (f_c + f_{c'}) bh \left[e - \frac{h}{6} \frac{(f_c - f_{c'})}{(f_c + f_{c'})} \right] \\ & - \alpha pbh f_{s'} \left[\frac{h}{2} - e - r'h \right] + pbh f_s \left[e + \frac{h}{2} - r'h \right] = 0 \end{aligned}$$

Substituting for $f_{c'}$ $f_{s'}$ and f_s , also $r'h = h - rh$

$$\frac{e}{2} \left(2 - \frac{1}{k} \right) - \frac{k}{12k} - \alpha pn \left(\frac{k+r-1}{k} \right) (rh - \frac{h}{2} - e) + pn \left(\frac{k-r}{k} \right) (e - \frac{h}{2} - rh) = 0$$

$$\frac{e}{2h} (2k-1) - \frac{1}{12} - \frac{1}{2} \alpha pn (k+r-1) (2r-1 - \frac{2e}{h}) + \frac{1}{2} pn (k-r) (\frac{2e}{h} + 2r-1) = 0$$

$$k \left[\frac{e}{h} - \frac{1}{2} \alpha pn (2r-1 - \frac{2e}{h}) + \frac{1}{2} pn (\frac{2e}{h} + 2r-1) \right]$$

$$= \frac{1}{2} \frac{e}{h} + \frac{1}{12} + \frac{1}{2} \alpha pn (r-1) (2r-1 - \frac{2e}{h}) + \frac{1}{2} pnr (\frac{2e}{h} + 2r-1)$$

$$k = \frac{1 + 6 \frac{e}{h} - 6 pn \left[(\alpha - \alpha r - r)(2r - 1) - 2 \frac{e}{h} (\alpha - \alpha r + r) \right]}{12 \frac{e}{h} - 6 pn \left[(\alpha - 1)(2r - 1) - \frac{2e}{h} (\alpha + 1) \right]} \quad (5)$$

From (4)

$$\begin{aligned} \frac{N}{bhfc} &= \frac{1}{2} \left(2 - \frac{1}{k} \right) + \alpha pn \left(\frac{k+r-1}{k} \right) + pn \left(\frac{k-r}{k} \right) \\ &= \frac{1}{2k} \left\{ (2k-1) + 2pn \left[(k-r) + \alpha (k+r-1) \right] \right\} \quad (6) \end{aligned}$$

When for symmetrical reinforcement $\alpha = 1$,

$$k = \frac{1 + 6 \frac{e}{h} (1 + 2pn) + 6pn (2r-1)^2}{12 \frac{e}{h} (1+2pn)}$$

and

$$\frac{N}{bhfc} = \frac{1}{2k} \left[(1 + 2pn)(2k - 1) \right]$$

$$\text{Diagram factor: } \frac{fcbh}{N} = \frac{fc}{\frac{N}{bh}} = fc \div \frac{N}{bh}$$

With assumed values for n , α , r , and p , Eq. (5) is an equation between k and $\frac{e}{h}$ and k can be found for any value of $\frac{e}{h}$.

If this value of k be substituted in Eq. (6), retaining the same values for n , α , r , and p , $fc \div \frac{N}{bh}$ can be determined and from this fc , since α , r , p , b , and h , are determined from the construction of the section and N and e are assumed to be known. Whence for each value of α a plate can be constructed consisting of a series of curves, one for each value of p , ordinates to the curve representing $fc \div \frac{N}{bh}$ and abscissae, $\frac{e}{h}$.

When the stress is wholly compressive, f_c is the only value that need be determined, for the steel stresses being n times those in the concrete, are below working values with working values in the concrete.

When k becomes less than unity there will be tension in the concrete, and its intensity can be obtained from Eq. (1), and the ratio $\frac{f_c'}{f_c}$ can be designated by a system of curves intersecting those above defined.

The reciprocal $\frac{f_c b h}{N}$ is used on the diagrams rather than $\frac{N}{f_c b h}$ as obtained from the equations, since the former gives straight lines when plotted against $\frac{e}{h}$. That this is so can be proven by substituting the value of k from Eq. (5) in (6)

$$\begin{aligned} \frac{b h f_c}{N} &= \frac{2k}{2k [1 + p n (\alpha + 1)] + 2 p n (\alpha - \alpha r - r) - 1} \\ &= \frac{1 + 6 \frac{e}{h} - 6 p n [(\alpha - \alpha r - r)(2r - 1) - 2 \frac{e}{h} (\alpha - \alpha r + r)]}{\{1 + p n (\alpha + 1)\} \{1 + 6 \frac{e}{h} [1 + 2 p n (\alpha - \alpha r - r)(2r - 1)] - 6 p n (\alpha - \alpha r + r)(2r - 1)\} - \{p n (\alpha + \alpha r + r) + \frac{1}{2}\} \{12 \frac{e}{h} [(1 + p n (\alpha + 1)) - 6 p n (\alpha - 1)(2r - 1)]\}} \\ &= \frac{1 + 6 \frac{e}{h} [1 + 2 p n (\alpha - \alpha r + r)] - 6 p n [(\alpha - \alpha r - r)(2r - 1)]}{[(1 + p n)(\alpha + 1)] [1 - 6 p n (\alpha - \alpha r - r)(2r - 1)] + [6 p n (\alpha - 1)(2r - 1)] [p n (\alpha - \alpha r + r) + \frac{1}{2}]} \end{aligned}$$

which is linear for $\frac{e}{h}$.

Case II.

No tensile resistance in concrete.

$$\frac{f_s'}{n f_c} = \frac{k + r - 1}{k} \quad \dots (2)$$

$$\frac{f_s}{n f_c} = \frac{r - k}{k} \quad \dots (3)$$

$$\begin{aligned} \frac{1}{2} f_c b k h + f_s' a p b h - f_s p b h \\ - N = 0 \quad \dots (7) \end{aligned}$$

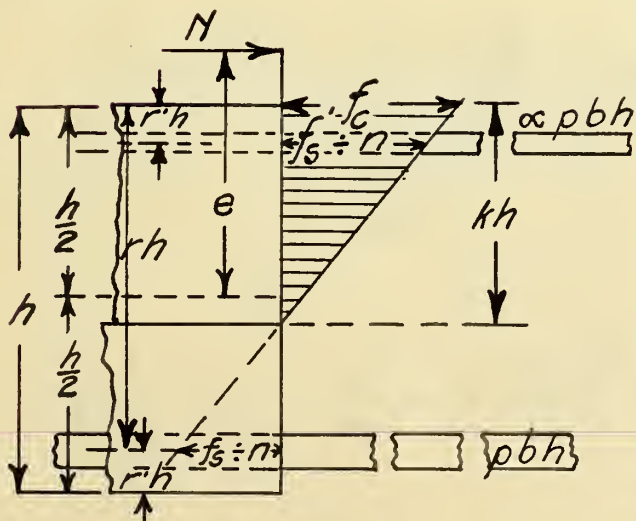


Fig. 3

$$\frac{1}{2} f_c b k h (e - \frac{1}{2} h + \frac{1}{3} k h) + \alpha p f_s' b h (e + \frac{1}{2} h - r h) - p f_s b h (e - \frac{1}{2} h + r h) = 0$$

Substituting for f_s' and f_s and reducing

$$k^3 + k^2 \left(\frac{3e}{h} - \frac{3}{2} \right) + 3pnk \left[(1-\alpha)(2r-1) + \frac{2e}{h} (\alpha+1) \right] \\ = 3pn \left[\alpha + r(2r-1) + \frac{2e}{h} (\alpha - \alpha r + r) - \alpha r(3-2r) \right] \dots \dots \dots (8)$$

From (7)

$$\frac{N}{bhfc} = \frac{1}{2}k + pn(\alpha+1) + \frac{pn}{k} (\alpha r - \alpha - r) \dots \dots \dots (9)$$

When $\alpha = 1$

$$k^3 + k^2 \left(\frac{3e}{h} - \frac{3}{2} \right) + 12pnk \frac{e}{h} = 3pn \left[\frac{2e}{h} + (2r-1)^2 \right]$$

$$\frac{N}{bhfc} = \frac{1}{2}k + 2pn - \frac{pn}{k} = \frac{1}{2}k + pn \left(2 - \frac{1}{k} \right)$$

When $\alpha = 0$

$$k^3 + k^2 \left(\frac{3e}{h} - \frac{3}{2} \right) + 3pnk \left(\frac{2e}{h} + 2r - 1 \right) = 3pnr \left(\frac{2e}{h} + 2r - 1 \right)$$

$$\frac{N}{bhfc} = \frac{1}{2}k + pn - \frac{pnr}{k} = \frac{1}{2}k - pn \left(\frac{r}{k} - 1 \right)$$

Diagram factor: $fc \div \frac{N}{bh}$

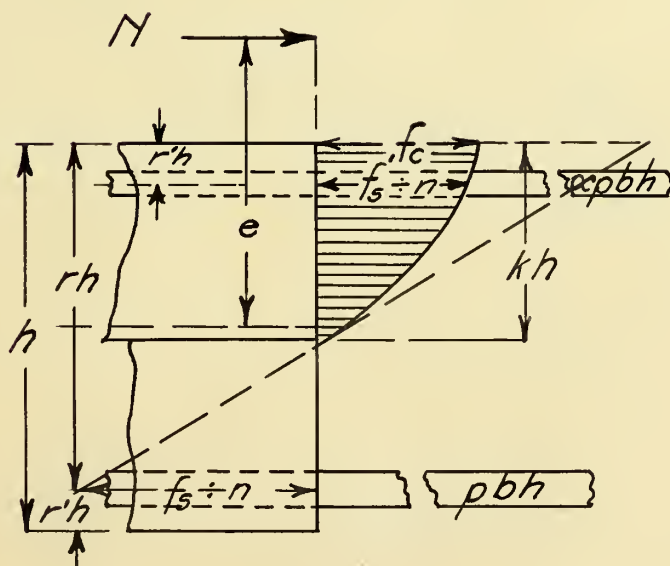


Fig. 4

Case III

Parabolic variation of compressive stress in the concrete.

$$\frac{f_s}{2nf_c} = \frac{r-k}{k} \dots \dots \dots (10)$$

$$\frac{f_s'}{2nf_c} = \frac{k+r-1}{k} \dots \dots \dots (11)$$

$$\frac{2}{3}fcbkh + fs'abh - fspbh - N = 0 \dots\dots\dots (12)$$

$$\frac{2}{3}fcbkh \left(e - \frac{h}{2} + \frac{3}{8}kh\right) + fs'apbh \left(e + \frac{h}{2} - rh\right) - fspbh\lambda$$

$$\left(e - \frac{h}{2} + rh\right) = 0$$

Reducing

$$k^3 + \frac{4}{3}k^2 \left(\frac{2e}{h} - 1\right) + 4pnk \left[(1-\alpha)(2r-1) + \frac{2e}{h}(\alpha+1)\right]$$

$$= 4pn \left[\alpha + 2r^2 - r + \frac{2e}{h}(\alpha - \alpha r + r) - \alpha r(3-2r)\right] \dots\dots\dots (13)$$

From 12

$$\frac{N}{bhfc} = \frac{2}{3}k + 2pn(1+\alpha) + \frac{2pn}{k}(\alpha r - \alpha - r) \dots\dots\dots (14)$$

When $\alpha = 1$

$$k^3 + \frac{4}{3}k^2 \left(\frac{2e}{h} - 1\right) + 16pnk \frac{e}{h} = 4pn \left[(2r-1)^2 + \frac{2e}{h}\right]$$

$$\frac{N}{bhfc} = \frac{2}{3}k + 2pn \left(2 - \frac{1}{k}\right)$$

When $\alpha = 0$

$$k^3 + \frac{4}{3}k^2 \left(\frac{2e}{h} - 1\right) + 4pnk \left(\frac{2e}{h} + 2r - 1\right) = 4pnr \left(\frac{2e}{h} + 2r - 1\right)$$

$$\frac{N}{bhfc} = \frac{2}{3}k - 2pn \left(\frac{r}{k} - 1\right)$$

$$\text{Diagram factor: } fc \div \frac{N}{bh}$$

Unsymmetrical Placement.

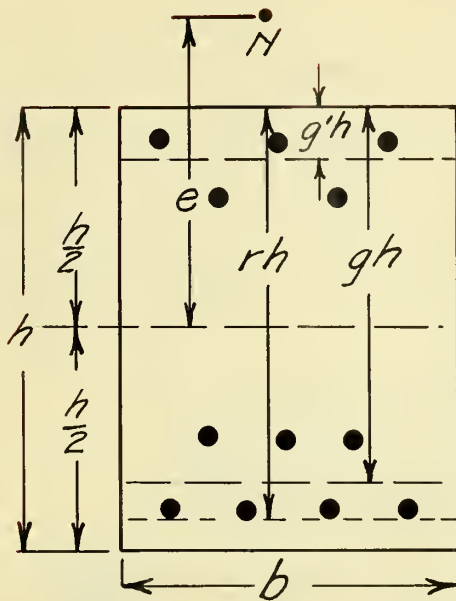


Fig. 5

The preceding equations apply only when the reinforcements are symmetrically placed so that their centers of gravity are respectively rh and $(1-r)h$ from the outer fiber. When the placing is unsymmetrical or the centers of reinforcements for each side are

in more than one plane, the preceding method may be applied by summing up the reinforcements on each side and finding the position of their respective centers of gravity. The summations are reduced to the form $p bh$ and $\alpha p bh$, and, designating by gh the distance from upper fiber to lower steel and $g'h$ the distance to the upper steel and applying the general method, the stress in the outer fiber of concrete is found, and from this the stress in the outer steel may be readily determined by considering proportionate distances from the neutral axis as expressed in Equations (1), (3), and (10).

If $g + g' = 1$, the equations deduced for symmetrical placement may be used. For the general case shown in Fig.5 with centers of gravity at distances gh and $g'h$ from the upper fiber, outer fiber of lower steel distant rh , the following general equations apply, $(n-1)$ being substituted for n to allow for the concrete displaced by the steel.

$$k = \frac{1 + 6\frac{e}{h} - 6p(n-1) \left[\alpha g' \left(1 - \frac{2e}{h} - 2g' \right) - g \left(\frac{2e}{h} + 1 - 2g' \right) \right]}{12\frac{e}{h} - 6p(n-1) \left[\alpha \left(1 - \frac{2e}{h} - 2g' \right) - \left(\frac{2e}{h} + 1 - 2g' \right) \right]} \quad (15)$$

$$\frac{N}{bhfc} = 1 - \frac{1}{2k} + p \frac{(n-1)}{K} \left[(k-g')\alpha + (k-g) \right] \dots \dots \dots (16)$$

$$\frac{fc'}{fc} = \frac{k-1}{k} \dots \dots \dots (1)$$

Case II

$$\begin{aligned} k^3 + k^2 \left(\frac{3e}{h} - \frac{3}{2} \right) + 3\alpha p(n-1)k \left(\frac{2e}{h} - 1 + 2g' \right) + 3pnk \left(\frac{2e}{h} + 2g - 1 \right) = \\ = 3\alpha p(n-1)g' \left(\frac{2e}{h} + 2g' - 1 \right) + 3png \left(\frac{2e}{h} + 2g - 1 \right) \dots \dots \dots (17) \end{aligned}$$

$$\frac{N}{bhfc} = \frac{1}{2}k + \alpha p(n-1) \left(1 - \frac{g'}{k} \right) + pn \left(1 - \frac{g}{k} \right) \dots \dots \dots (18)$$

$$\frac{fs}{nfc} = \frac{r-k}{k} \dots \dots \dots (3)$$

Case III

$$\begin{aligned} k^3 + k^2 \left(\frac{8}{3} \frac{e}{h} - \frac{4}{3} \right) + 4\alpha p(n-1)k \left(\frac{2e}{h} - 1 + 2g' \right) + 4pnk \left(\frac{2e}{h} + 2g - 1 \right) = \\ = 4\alpha p(n-1)g' \left(\frac{2e}{h} + 2g' - 1 \right) + 4png \left(\frac{2e}{h} + 2g - 1 \right) \dots \dots \dots (19) \end{aligned}$$

$$\frac{N}{bhfc} = \frac{2}{3}k + 2\alpha p(n-1) \left(1 - \frac{g'}{k} \right) + 2pn \left(1 - \frac{g}{k} \right) \dots \dots \dots (20)$$

$$\frac{fs}{2nfc} = \frac{r-k}{k} \dots \dots \dots (10)$$

MATERIALS, TEST PIECES AND TESTING

Materials.— The test pieces were made in the Concrete Laboratory of the University of Illinois. The aggregate consisted of Kankakee limestone and Attica sand. The proportion of sizes and fineness are shown for each in Tables 1 and 2, the figures given being the average of two samples tested.

Table 1.

Analysis of Stone

Size of Mesh	Percent Passing
1 in.	100
3/4 in.	85.5
1/2 in.	47.6
3/8 in.	21.4
No. 3 sieve	7.8
No. 5 sieve	.8
No. 10 sieve	.4

Table 2.

Analysis of Sand

Size of Mesh	Percent Passing
No. 3 sieve	100
1/4 in.	99.1
No. 5 sieve	95.7
No. 10 sieve	70.0
No. 12 sieve	62.1
No. 16 sieve	53.9
No. 18 sieve	43.5
No. 30 sieve	25.3
No. 40 sieve	13.3
No. 50 sieve	6.4
No. 74 sieve	3.8
No. 150 sieve	1.0

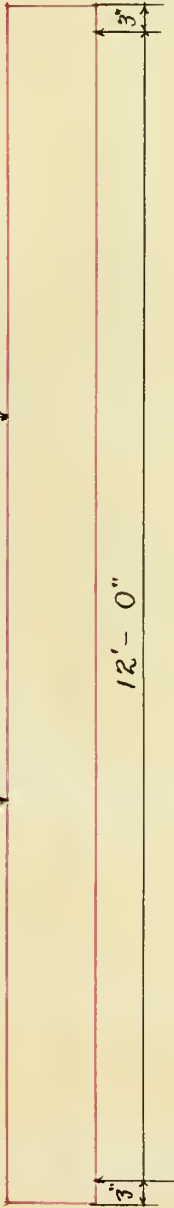
Universal portland cement, manufactured by The Universal Portland Cement Co. of Chicago, was used. The strength in tension is shown in Table 3, each value being an average of five determinations.

Table 3.

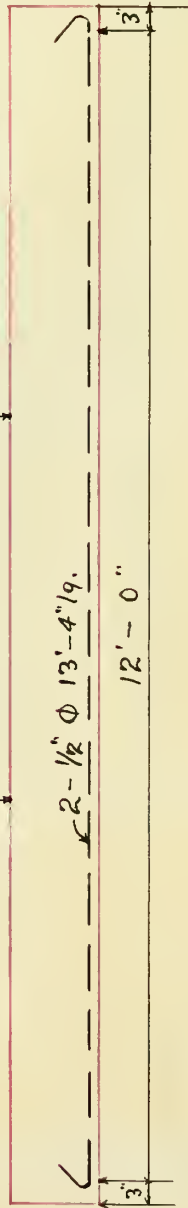
Tensile Strength of Cement				
Ref. No.	Ultimate strength, lb. per sq. in.			
	Age 7 days		Age 28 days	
	Neat	1-3 Mortar	Neat	1-3 Mortar
1	670	190	697	297
2	629	219	678	328
3	613	217	649	315
Av.	637	209	675	313

Test Pieces.— Twenty-three test beams were made, of which twenty-two were tested. Table 4 gives dimensions and reinforcements, and Fig. 6 shows the design. In proportioning the aggregate one bag of cement was taken to be one cubic foot, the corresponding amounts of sand and stone being two cubic feet and four cubic feet, respectively. Beams No. 450.1 to 450.5 are plain concrete. Beams No. 451.1 to 451.5 have about .44% bottom reinforcement. Beams No. 452.1 to 452.5 have .44% bottom reinforcement and .88% top reinforcement. Beams No. 453.1 to 453.4 have about 1.0% bottom reinforcement and Beams No. 454.1 to 454.4 have 1% reinforcement in top and bottom, respectively. These amounts were chosen to bring out the effects of different ratios of tensile and compressive re-

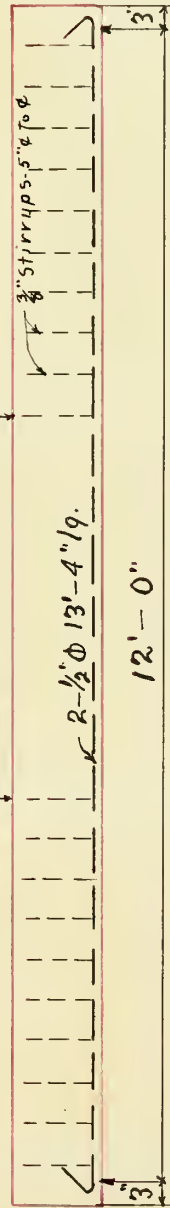
5 Beams 450.1 to 450.5



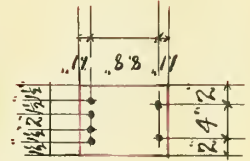
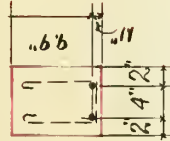
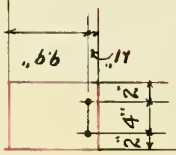
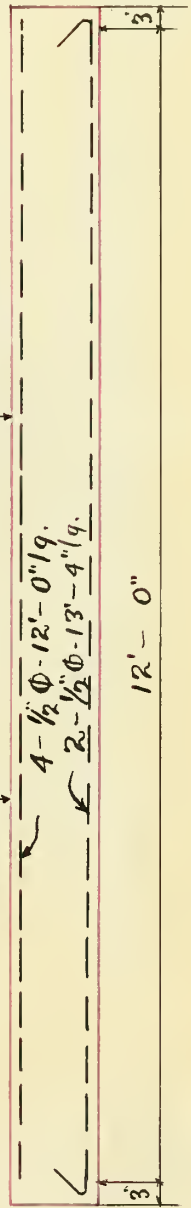
3 Beams 451.1, 451.2 and 451.5



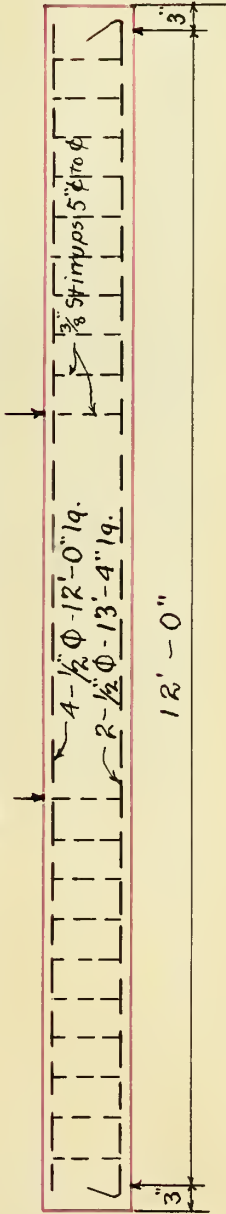
2 Beams 451.3 and 451.4



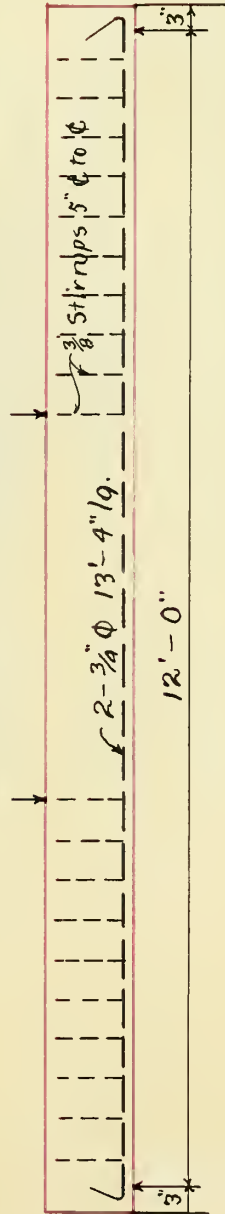
3 Beams 452.1, 452.2 and 452.5



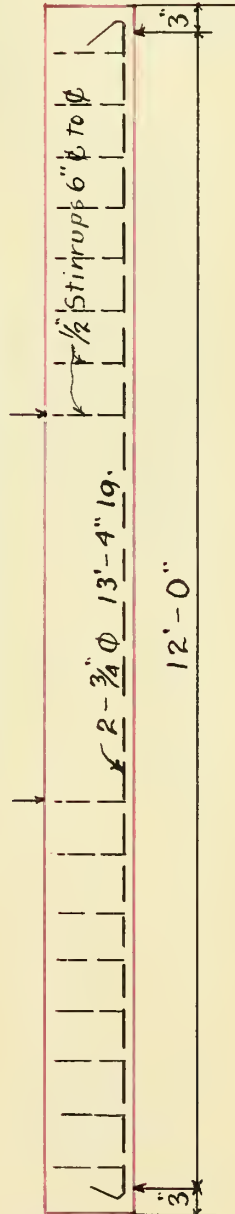
2 Beams 452.3 and 452.4



2 Beams 453.1 and 453.2



2 Beams 453.3 and 453.4



4 Beams 454.1 to 454.4

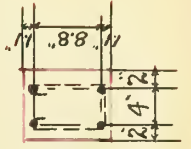
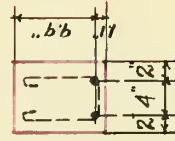
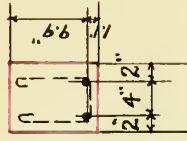
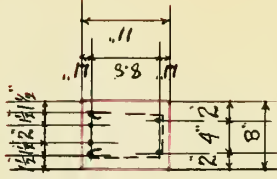
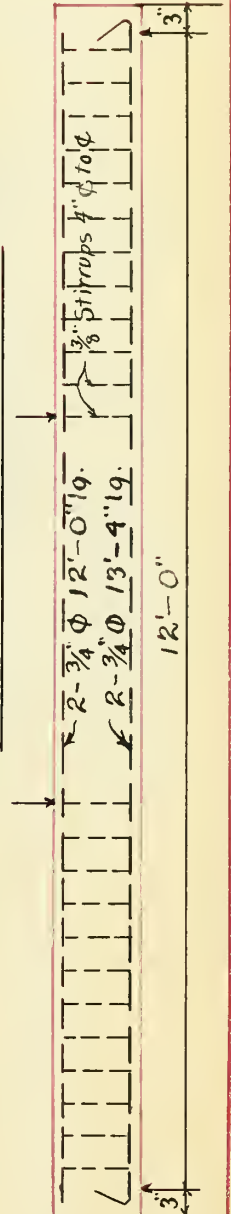


Table 4.

Data of Beams

All beams 12 ft. 6 in. long. Concrete 1-2-4

Beam No.	Section Concrete	Percent Cement	Age days	Percent Steel		Proportionate Distance to		Web Reinforcement	
				top	bottom	top	bottom	per-cent	disposition
450.1	8.187x11.000	12.2	67	Plain concrete					
450.2		15.3		Plain concrete, broken in hauling.					
450.3	8.125x11.125	15.8	70	Plain concrete					
450.4	8.06 x11.31	15.0	69	Plain concrete					
450.5	8.12 x11.125	14.6	70	Plain concrete					
451.1	8.25 x11.312	13.2	63	0	.40	0	.882	None	
451.2	8.00 x11.062	15.4	70	0	.447	0	.888	None	
451.3	8.00 x11.00	13.2	65	0	.450	0	.900	.482	Ten ³ / ₈ in.
451.4	8.00 x11.5	15.0	69	0	.443	0	.903	.465	spaced
451.5	(8 x 11)	14.8	69	0	.438	0	.900	None	5 in.
452.1	8.12 x11.37	13.1	63	.856	.428	.132	.812	None	
452.2	8.06 x11.37	15.3	70	.882	.441	.129	.900	None	
452.3	8.00 x11.12	15.1	69	.882	.441	.173	.895	.465	do.
452.4	8.00 x11.375	14.9	70	.862	.431	.151	.890	.470	do.
452.5	8.06 x11.5	15.1	69	.846	.423	.125	.86	None	
453.1	8.25 x11.25	14.9	70	0	.950	0	.89	.460	do.
453.2	8.12 x11.37	14.9	71	0	.952	0	.88	.465	do.
453.3	8.00 x11.25	14.6	71	0	.982	0	.89	.722	8 ¹ / ₂ in. spaced
453.4	8.00 x 11.25	14.5	71	0	.975	0	.872	.57	12 ⁵ / ₈ in.
454.1	8.00 x 11.25	15.0	70	.975	.975	.105	.885	.57	spaced
454.2	7.75 x 11.12	14.7	71	1.025	1.025	.125	.900	.595	4 in.
454.3	8.12 x 11.43	14.9	70	.945	.945	.125	.880	.550	do.
454.4	7.87 x 11.00	15.0	72	1.022	1.022	.119	.895	.590	do.

inforcements with the methods of loading to be described later. To prevent a possible failure by diagonal tension, web reinforcement sufficient to take two thirds of the shear was placed in all beams in which the shear at the calculated maximum load exceeded 50 lb. per sq. in.

To check the quality of the concrete three 6-in. cubes and one control beam 6 x 8 in. x 3 ft. 4 in. were made from the same batch as the corresponding beam, and also for the greater number of beams, one 8 x 16 in. cylinder was made.

Method of Making.— The beams were made in the standard wooden forms of the Laboratory, the cubes in triple forms made of cast iron, and the cylinders in steel forms. The mixing was done by hand, the cement and sand being first mixed dry, after which the stone was added and mixed. Water was then added and the whole mixed to uniform consistency, the amount of water varying from 8% to 9% by weight.

Storage.— The beams were stored in air under an average temperature of 60° F. and were sprinkled once each day. The cubes were stored in damp sand, and the cylinders were stored in air but were kept a little more damp than the beams so that their rate of drying may be expected to approximate that of the beams, if the larger size of the latter is considered.

Testing Apparatus.— The testing apparatus was arranged so that the thrust and bending moment at all stages of the test would be constant over the whole gage length. To affect this, the beams were placed on supports, forming a span of 12 ft., on the bed of the 200 000-lb. Olsen testing machine, a vertical

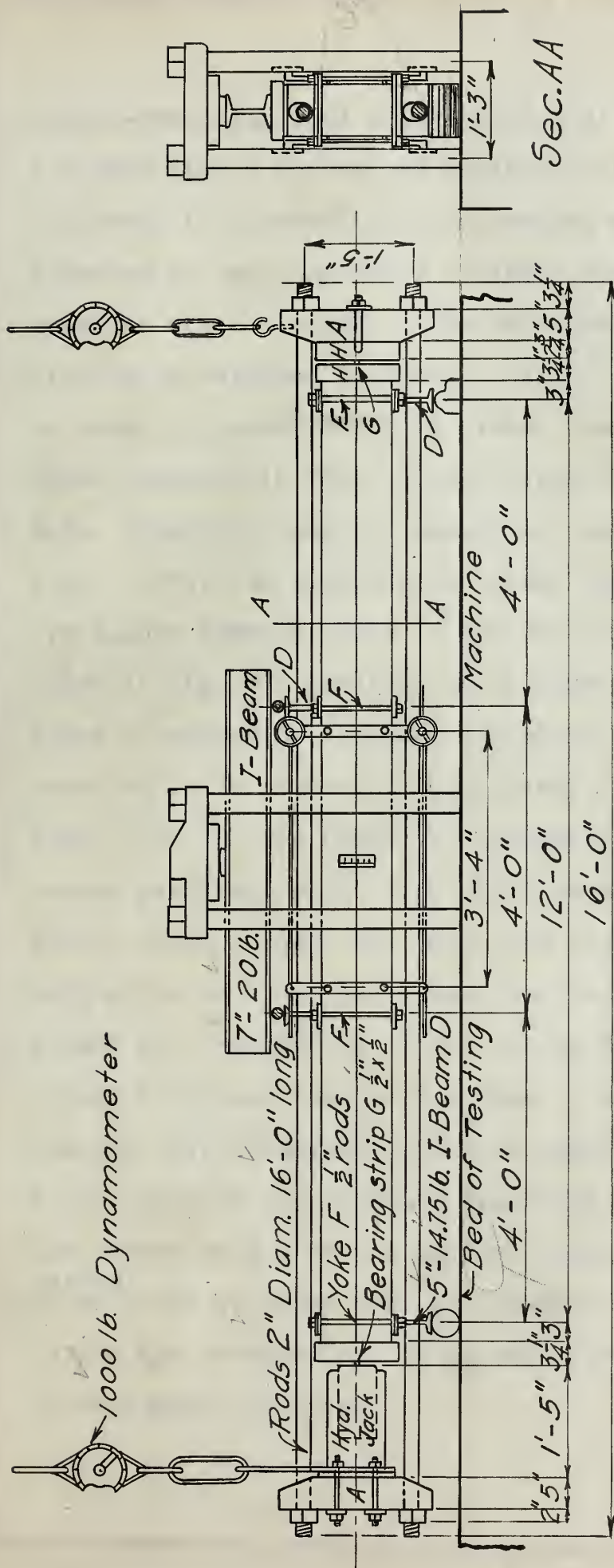


Fig. 7

moment-producing load being applied at the one-third points. At the same time a thrust was applied horizontally at the ends of the beams by a hydraulic jack working against two blocks tied together by two rods which extended along the beam and took the reaction of the thrust. The arrangement of the apparatus as originally designed is shown in Fig. 7 and the changes made may be noted by comparison with Views 1 and 2. The blocks "A", shown separate in Fig. 8, have cored holes through which the 2-in. retaining rods are passed and secured by nuts on the outside. Over the supports and under the load points the rods are passed through slots in the webs of 5-in. I-beams, cut as shown in Fig. 9. Between the flange and the beam is placed a piece of rubber belting and the whole is secured in place by means of bolts passing through straps bearing against the opposite face. To the one block the hydraulic jack is bolted in a horizontal position, and to the other an auxiliary block "H". The whole, blocks, rods, and jack, are supported from above and during the test are kept free from the rest of the apparatus except for the bearing at the strips "G" through which the thrust is transmitted to the beam. As a deflection of the machine bed, consequent upon the application of vertical load, would increase the overhead reactions and thus interfere with the determination of the applied load, a spring dynamometer is placed on each end as shown, and its reading kept constant by manipulating the turnbuckles, or as was found to be more convenient, by adjusting the hoist.

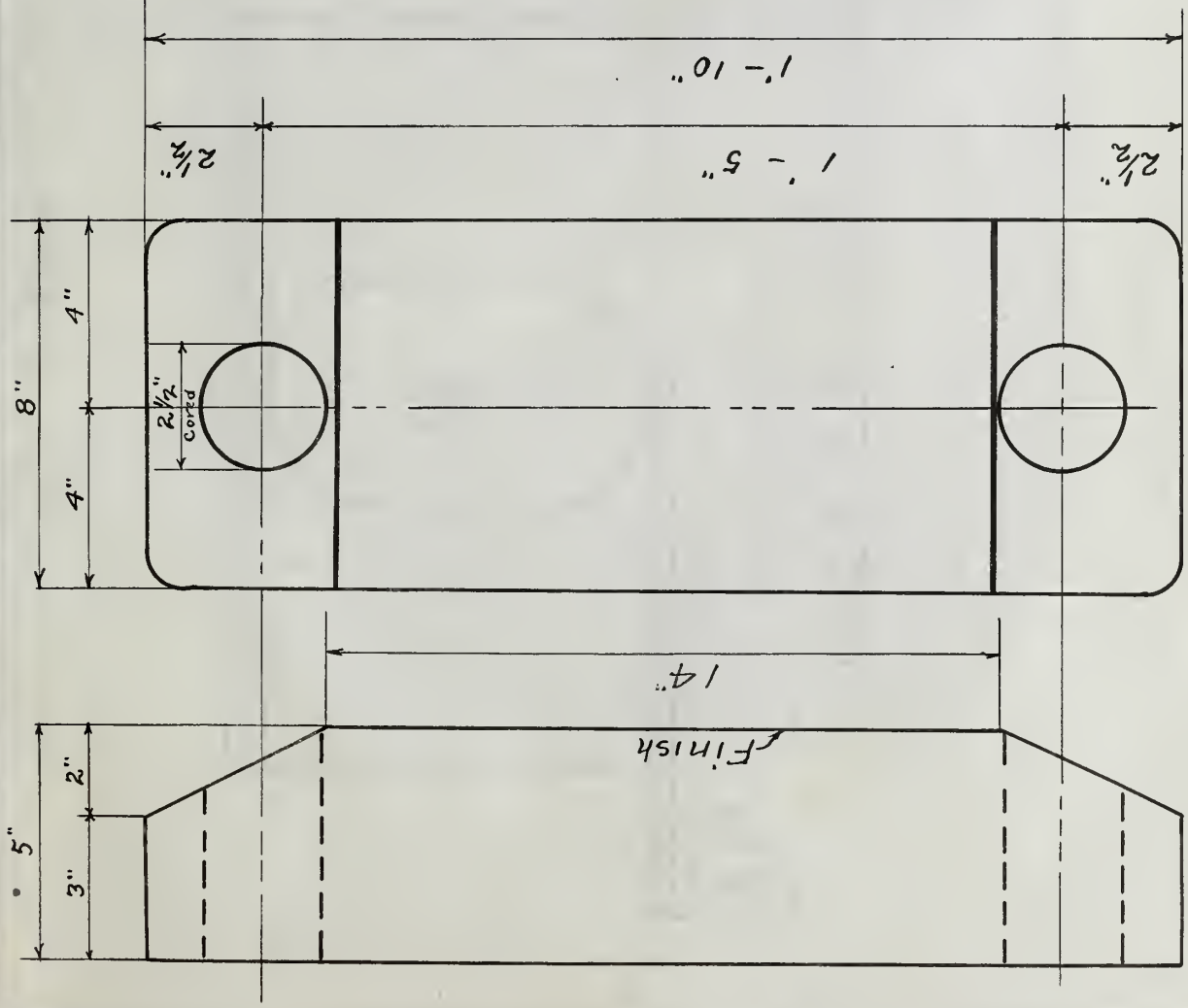


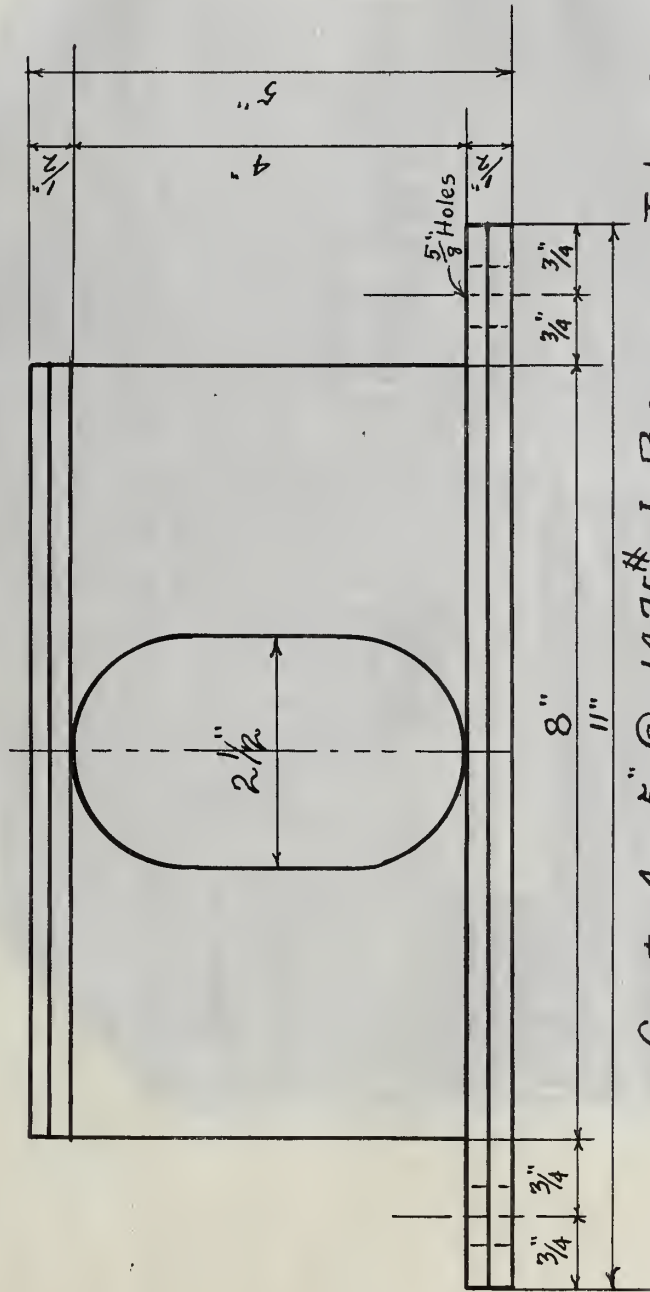
Fig 8.

Make 2 C.I. Blocks Thus

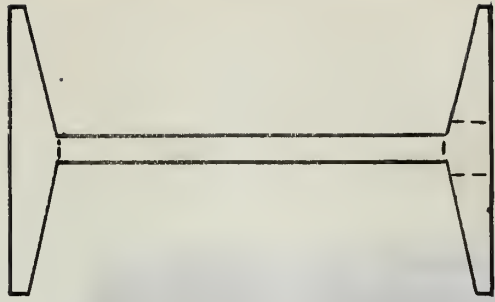
Designed for 2 Load ea. 25 Tons
 Applied 17" Apart
 With Inner Bearing Surface 12" lg.

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Fig. 9.



Cut 4 - 5" @ 14.75# I Beams Thus

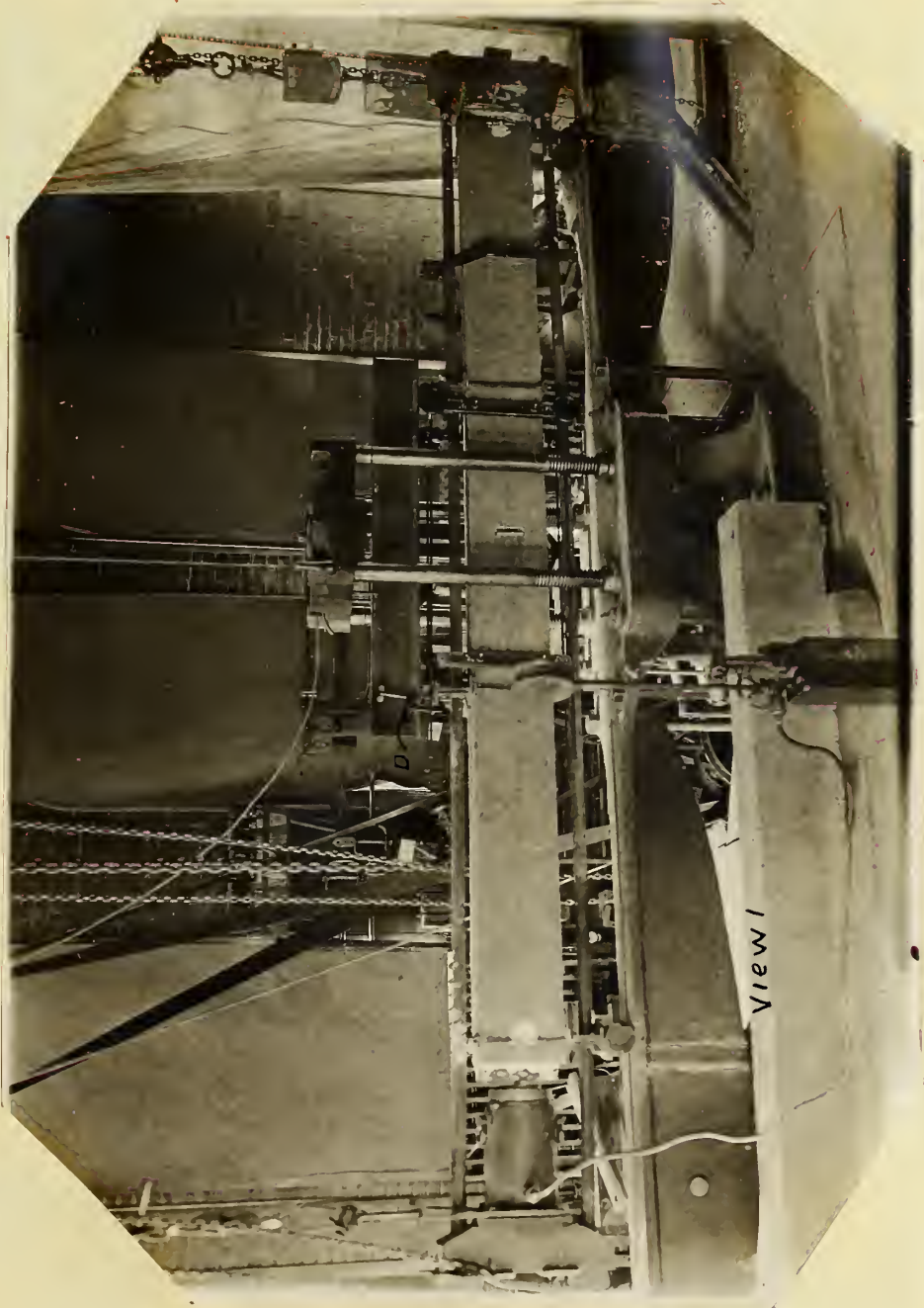


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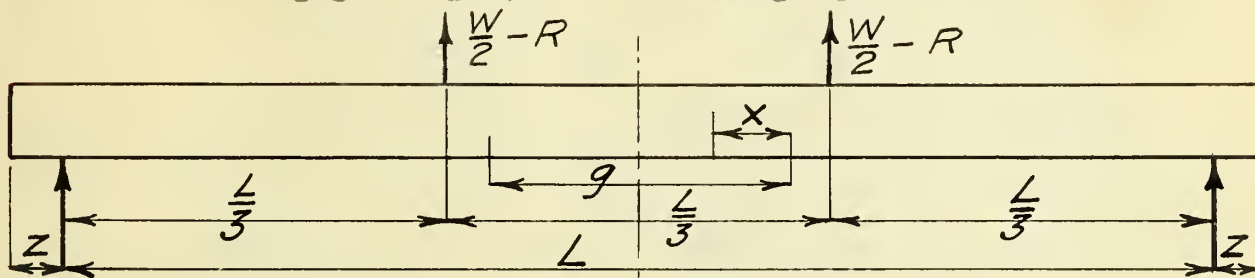
Over the ends of the beam the bearing blocks "H" are bedded in plaster of paris and on them and central with the height of the beam are placed the $\frac{1}{2} \times \frac{1}{2}$ -in. bearing strips "G". One vertical reaction is fixed and one is free, as shown in View 1, and the vertical load is applied through two rollers. Pressure is supplied to the jack by oil under pressure from the pump whose connections are shown in Views 1 and 2. For registering the lower horizontal loads the 500-lb. hydraulic gage was used, and for the higher loads the regular jack gage No. 848. The calibration of these two gages is shown in Diagrams 1 and 2, respectively.

Deformations were measured on a 40-in. gage length by means of the regular beam extensometers of the station, the upper roller being placed $5\frac{1}{2}$ in. above the beam. Deflections were read by means of a thread, fixed to the center of the beam above the supports, and passing over a mirror scale at the center.

Details of Test.— Since the stress on a section subject to thrust and bending is a function of the intensity of the thrust and ratio of moment to thrust, it becomes necessary in order to afford means for theoretical comparison to begin the taking of deformations from the point of zero moment as well as of zero thrust. It thus becomes necessary to counter-balance the dead load moment of the beam at the beginning of the test to produce zero moment in the gage length. To accomplish this a method similar to that described in Bulletin No. 329 of the U. S. Geological Survey was employed. By means of two clamps, one at each one-third point, as shown in View 1 at D,



the loading I-beam, which is screwed to the head of the machine by means of four bolts passing through holes in its flanges, is secured to the load stirrups F so that an upward force may be exerted at the one-third points by running the machine head up. To determine the amount of uplift necessary to produce zero moment in the gage length, the following equations were deduced:



If L = distance between supports

g = gage length which is taken symmetrical with the center

z = overhanging of beam at each end

W = total weight of beam

R = reaction at each end

Then

$\frac{W}{2} - R$ = force exerted by each stirrup at distance $\frac{L}{3}$ from supports.

m = constant bending moment over the gage length due to load stirrups, blocks, etc.

Then taking origin of coordinates at one end of the

gage length

$$M = R\left(\frac{L}{2} - \frac{g}{2} + x\right) + \left(\frac{W}{2} - R\right)\left(\frac{L}{6} - \frac{g}{2} + x\right) - \frac{W}{2(L + 2z)}\left(\frac{L}{2} + z - \frac{g}{2} + x\right)^2 + m$$

Which reduces to

$$M = \frac{RL}{3} - \frac{W}{4}\left(\frac{L}{6} + z\right) - \frac{W}{4\left(\frac{L}{2} + z\right)}\left(\frac{g}{2} - x\right)^2 + m$$

Then from the condition $\int_0^{\frac{g}{2}} M dx = 0$

$$\left[\frac{RL}{3}x - \frac{W}{4}\left(\frac{L}{6} + z\right)x - \frac{W}{4\left(\frac{L}{2} + z\right)}\left(\frac{g^2}{4}x - \frac{1}{2}gx^2 + \frac{x^3}{3}\right) + mx \right]_0^{\frac{g}{2}} = 0$$

Substituting $x = \frac{g}{2}$ and reducing

$$R = \frac{3}{4} \frac{W}{L} \left(\frac{L}{6} + z \right) + \frac{Wg^2}{16L \left(\frac{L}{2} + z \right)} - \frac{3m}{L}$$

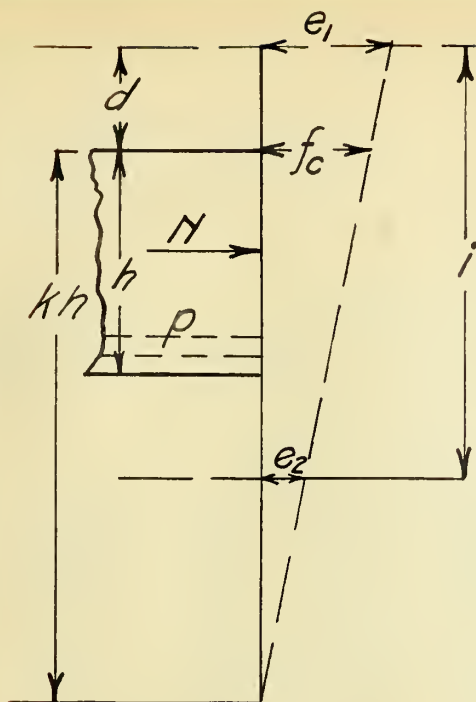
which is the value of each reaction when the bending moment in the gage length is zero.

Substituting $L = 144$ in., $z = 3$ in., and $g = 40$ in.,

$$R = .14977W + \frac{1}{48}m.$$

The positive moment produced by the load stirrups, each 16 lb. in weight, nearly equals the negative moment produced by the end bearing blocks, each 110 lb. in weight, and as the weight of the extensometers more than balances the difference, m may be taken equal to zero and hence $2R = .3W$ is the load on the machine due to the beam when the bending moment in the gage length is zero. To this must be added the weight of supports and end blocks to get the desired machine reading.

At the beginning of the test the end bearing strips were placed as nearly on the geometric center as could be done by measurement. This was checked by reading the extensometers after the end load was applied. It is evident that, for the plain beams and for those symmetrically reinforced, top and bottom, the sum of the top extensometer readings should equal that of the bottom ones. For those unsymmetrically reinforced, as beams No. 451, 452, and 453, the ratio of the two extensometer readings may be obtained from theoretical considerations as follows:



Given a beam unsymmetrically reinforced as shown, with extensometers distant i apart and upper roller distance d above beam. Find from Equation 5, preceding, the depth of the neutral axis kh due to the load N on the geometric center of the section ($\frac{e}{h} = 0$). Then the ratio of extensometer readings $e_1 : e_2$ may be obtained by considering pro-

portionate distances from the neutral axis. $\frac{E_s}{E_c} = n$ is taken at a low value which can be approximately checked by considering the deformation obtained. By these means the following required ratio ($e_1 : e_2$) of extensometer readings was found for $N = 10$:

Beams No. 451.1-5, $e_1 : e_2 = 1.35$

Beams No. 452.1-5, $e_1 : e_2 = .707$

Beams No. 453.1-4, $e_1 : e_2 = 1.98$

It was observed, however, that the lower fiber seemed stiffer than the upper, due possibly to a greater amount of water present during the drying process so that the above requirements were not in all cases strictly adhered to.

The end thrust was kept as nearly constant as practicable during the whole or a part of the run. In the tests in which it was desired to cause failure by compression, the thrust was increased by successive stages to produce about 300, 600, 800,

and sometimes 900 lb. per sq. in. of section. As an illustration the log of test of Beam No. 453.3 is reproduced below.

Beam 453.3 Tested Feb. 19, 1910

Machine	Upper		Lower		End	Center	Notes
Load	Extensometers	Extensometers	Extensometers	Extensometers	Thrust	Deflec-	
	1	2	3	4	Gage	tion	
						Scale Wt. (Est.)=1230	
719	.0000	.0000	.0000	.0000	0	1.63	.3W+350=719
719	+.0055	.0041	-.0020	-.0031	15 tons	1.64	← Required $\frac{e_1}{e_2}=1.98$
1 600	.0093	.0073	+.0008	-.0006	do.	1.65	
2 200	.0117	.0096	.0026	+.0011	do.	1.67	
3 000	.0152	.0130	.0054	+.0035	do.	1.68	
4 000	.0193	.0175	.0088	.0070	do.	1.70	
5 000	.0245	.0225	.0129	.0112	do.	1.73	
6 000	.0305	.0283	.0180	.0163	do.	1.76	
7 000	.0378	.0353	.0244	.0228	do.	1.80	
8 000	.0464	.0435	.0323	.0314	do.	1.84	
9 000	.0559	.0545	.0412	.0413	do.	1.88	
10 000	.0658	.0613	.0497	.0513	do.	1.93	
11 000	.0759	.0710	.0590	.0615	do.	1.98	
9 100	.0860	.0770	.0529	.0504	29 tons	2.00	
10 000	.0915	.0828	.0567	.0548	do.	2.02	
11 000	.0975	.0885	.0620	.0603	28.5 tons	2.05	
12 000	.1070	.0973	.0690	.0676	29 tons	2.10	
13 000	.1163	.1059	.0764	.0759	do.	2.14	
14 000	.1290	.1175	.0860	.0860	do.	2.20	
15 000	.1419	.1297	.0972	.1025	do.	2.26	Fine tension
13 200	.1528	.1365	.0933	.0875	36.5 tons	2.26	and diagonal
14 000	.1655	.1482	.1025	.0952	38 tons	2.31	cracks 3 in.
15 000	.1809	.1628	.1130	.1045	37.5 tons	2.38	long.
16 000	.2044	.1854	.1300	.1197	do.	2.46	
16 100	.2480	.2390	.1770	.1440	36.0 tons	2.60	Compression failure 8 in. inside S. load point.

On increasing the end thrust the vertical load usually fell off, but recovered again rapidly on starting the machine. The deformations obtained at these points do not indicate as much drop in load as the scale beam indicated, so it seems unnecessary to consider these back readings as a part of the run.

Table 5.

Auxiliary Tests

Beam No.	Compressive Strength of 6 in. cubes		Compressive Strength of 8 x 16 in. cylinder		Modulus of Rupture Control Beam		Yield Point of Steel		Ultimate Strength of Steel Bottom lb.per sq.in. Av.of 2
	Age	Av.of 3 lb. per sq. in.	Age	lb. per sq. in.	Age	lb.per sq.in.	Top lb. per sq. in. Average of 4	Bottom lb. per sq. in. 2	
450.1	68	2180	70	1135					
450.3					73	448			
450.4					73	364			
450.5									
451.1	66	2410	70	1564	65	293		33000	53400
451.2	71	2630	71	2405	71	368		36500	65100
451.3	66	1860	66	1650	66	295		36900	57500
451.4	72	2570	72	1988	72	326		34800	63150
451.5	70	1965	72	2025	69	333		37000	66600
452.1	68	1980	69	1150	67	294	32150	38600	67350
452.2	72	2255	72	2055	72	395	34300	34800	51200
452.3	69	2710	69	1911	71	339	36350	34800	52300
452.4	72	2590*	73	1890	73	397	37700	34500	50050
452.5	70	2430	70	2025	70	267	35900	37900	62600
							Av.	35880	58940
453.1	71	2630	73	1980	71	325		41870	60950
453.2	70	2400			74	325		42500	60500
453.3	71	2230	73	1562	71	398		40100	58150
453.4	72	2140			72	320		42000	61350
454.1	70	2650*	72	1057	70	357		42000	60250
454.2	71	2590*			70	342		42450	61600
454.3	71	2720 ⁺	70	1779	71	374		40300	60400
454.4	75	2620 ⁺			74	345		41000	59750
Av.		2330		1740		345		41527	60370

*Capacity of machine, 2800 lb. per sq. in., exceeded by one cube

⁺Capacity of machine, 2800 lb. per sq. in., exceeded by two cubes

Average initial modulus of elasticity of concrete = 2270000 lb. per sq. in.
 Modulus of elasticity of steel = 29000000 lb. per sq. in. [sq. in.]



View 3

Auxiliary Tests.— Compression tests were conducted on the 6-in. cubes and 8 x 16-in. cylinders. The tensional strength of the concrete in the control beams was determined by testing them on a 36-in. span under one-third point loading. The yield point and ultimate strength of the reinforcing steel was determined in tension tests. At the same time the modulus of elasticity of steel in tension and of the concrete in compression were also obtained. The cubes were bedded in plaster of paris out of the machine by means of glass plates laid as nearly perpendicular to the sides of the cubes as possible with the help of level and square. The cylinders were prepared for the test in the same way. In testing, one or more thicknesses of building paper was put between the head and bed of the machine and faces of the cubes, and on top was placed a spherical bearing block carefully centered, both with reference to the cube and the line of applied load. To obtain deformations on the cylinder, the deformation frames shown in View 3 were used, the deformations being indicated on the wire-wound dial. The frames are pivoted by the contact points across a diameter, deformations being thus obtained over twice the gage length of 10 in., the dial reading one half of the deformation.

The modulus of elasticity of the steel was obtained by measuring extensions with a Ewing extensometer.

The computed results for the auxiliary tests are given in Table 5. Diagram 3 gives the average modulus of elasticity of ten test cylinders plotted against the unit deformation. The

use of this diagram to obtain the modulus of elasticity of concrete in the beam tests is shown in Diagram 4, and will be discussed later. Following Diagram 4 are the experimental plots of the moduli of elasticity of the individual cylinders against the corresponding unit deformations and unit stresses.

COMPUTATIONS ON TEST DATA.

Reduction of Experimental Data.— The quantities obtainable from the extensometer readings are position of neutral axis and unit deformation in steel and outer fiber of concrete. To facilitate the determination of these quantities Diagrams A and B have been prepared.

Assuming the upper roller distant d above the beam and the rollers distant i apart vertically, referring to diagram on page 29, we have

$$\frac{e_1}{Kh + d} = \frac{e_1 - (-e_2)}{i}$$

e_1 = average of upper extensometer readings

e_2 = average of lower extensometer readings

Solving and making $i = 20.312$ and $d = .515$

$$\frac{e_1}{e_2} = \frac{.2705 + .0492 Kh}{.7295 - .0492 Kh}$$

Diagram A is a plot of this equation. From this diagram, Kh , and therefore K , can be obtained for given values of e_1 and e_2 . Also, if e_c = unit deformation of concrete in the top fiber of the beam,

$$e_c = \frac{e_1}{40} \times \frac{Kh}{Kh + d}$$

Substituting for K_h from preceding equation and making $d = 5.5$ in. and reducing,

$$e_c = .01822 e_1 - .006765 e_2.$$

Diagram B is the corresponding plot for this equation. The unit deformation in ^{the} steel, tensile and compressive, is obtained with reference to e_c by considering proportionate distances from the neutral axis.

As the object of these experiments was to compare theory with fact and as the values obtained from theory are expressed as stresses in the materials, it seemed desirable to reduce the unit deformations obtained as above to stresses. In the case of steel this could be done by multiplying by the modulus of elasticity, which is practically constant up to the elastic limit and approximately so up to the yield point. With regard to the concrete the only information available as to its modulus is the average modulus-deformation plot of Diagram 3. As seen, the average unit deformation of the ten cylinders considered, at maximum stress, is .0016, at which point the modulus is one half ^{of} the initial modulus, or 1 135 000 lb. per sq. in. This gives a stress of 1822 lb. per sq. in. The maximum deformation in the outer fiber of the beams is obtained from the following beam data on compression or near compression failures.

Beam No.	Maximum Load	Load Considered	Unit Deformation
451.5	15 200	15 000	.002435
453.3	16 100	16 000	.00266
453.4	18 000	17 000	.00260
452.4*	17 600	17 500	.00200
454.3*	22 300	22 300	.00225

*Tension failure followed by compression failure.

Considering the above, .0026 seemed a correct value to adopt for the maximum unit deformation in beams, and assuming stress and deformation to be related by the parabolic law up to this point, the modulus of elasticity would be one half of the initial value. This principle has been applied in Diagram 4, the form of the experimental modulus-deformation curve of Diagram 3 has been retained, and the vertex of the stress-deformation parabola located at unit deformation .0026 by giving the modulus at this point the value 1 135000, one half of the initial value, 2 270 000.

The assumption of parabolic variation for the stress-deformation relation in the outer fiber of the beam will give values of the stresses as high or higher than the actual values, as the curve for direct compression near the vertex is somewhat fuller than the parabola, a tendency that would be further accentuated in the beam on account of the interaction of fibers unequally stressed.

The dead weight of the beam together with the moment produced by the end thrust acting with the average deflection in the gage length as lever arm were reduced to an equivalent moment load applied at the one-third points, for purposes of plotting. To reduce the weight of the beam to equivalent one-third point loading, the moment M for uniform load was integrated with respect to x over one half of the gage length, and this integral divided by the number of units of distance in one half of the gage length gives the average ordinate to the moment curve in terms of WL . The coefficient of WL thus obtained divided by the coefficient for one-third point loading gives the fraction by which the weight of the beam must be multiplied to reduce it to equivalent one-third point loading. Thus, for uniform load

$$M = \frac{1}{2}wlx - \frac{1}{2}wx^2$$

$$\int_{0.611}^{.61} Mdx = \left[\frac{1}{4}wlx^2 - \frac{1}{6}wx^3 \right]_{.611}^{.61} = .01698 \, wl^3$$

$$.01698 \, wl^3 \div .139 \, l = .122 \, Wl.$$

For one-third point loading $M = \frac{1}{6} Pl$

$$\therefore \text{Required coefficient} = \frac{.122}{.1666} = .732$$

To find the one-third point loading that will produce the same moment as the end thrust acting on the deflected center sections, the average deflection in the gage length was obtained by integrating the equation of the elastic curve for one-third point loading in a manner similar to that of the above described moment curve.

Table 6.

Experimental Values for Beam 453.3

Machine Load	Equivalent $\frac{1}{8}$ Pt. Load	Cage Reading	Total Thrust $x \frac{26}{24}$	Center Deflec- tion in.	$\frac{1}{4}$ Pt. Total Load	Experimental Values			
						K	e_c	e_s	f_c f_s
719	000	15.0 tons	26400	.01	10	3.360	+.000104	+.0000768	252 +2226 wt=1230
1600	900	do.	"	.02	21	1.30	.000150	+.0000473	348 +1370 1230 x
2200	1500	do.	"	.04	42	1.05	.000200	+.0000305	448 + 885 .732 =
3000	2300	do.	"	.05	52	.884	.000230	-.0000015	- 43 900.
4000	3300	do.	"	.07	73	.780	.000280	-.0000395	600 -1145 Section
5000	4300	do.	"	.10	105	.708	.000345	.0000888	722 -2570 = 11 $\frac{1}{2}$ x
6000	5300	do.	"	.13	136	.662	.000415	.000143	847 -4150 8 in.
7000	6300	do.	"	.17	178	.615	.000510	.000228	1010 6620 = 90
8000	7300	do.	"	.21	220	.570	.000600	.000337	1152 9770 r= .89
9000	8300	do.	"	.25	262	.545	.000725	.000458	1350 13300
10000	9300	do.	"	.30	315	.518	.000815	.000585	1470 17000
11000	10300	do.	"	.35	368	.506	.000935	.000710	1630 20600
9100	8400	29.0 tons	52800	.37	782	.620	.001140	.000496	1870 14400
10000	9300	do.	"	.39	825	.615	.001210	.000542	1950 15720
11000	10300	28.5 tons	52000	.42	888	.603	.001285	.000612	2030 17750
12000	11300	29.0 tons	52800	.47	995	.598	.001410	.000689	2130 19950
13000	12300	do.	"	.51	1080	.587	.001610	.000830	2340 24100
14000	13300	do.	"	.57	1205	.578	.001670	.000902	2390 26200
15000	14300	do.	"	.63	1332	.552	.001840	.001130	2520 32800
13200	12700	36.5 tons	67500	.63	1700	.627	.002040	.000857	2650 24900
14000	13300	38.0 tons	70000	.68	1957	.624	.002180	.000930	2720 27000
15000	14300	37.5 tons	69000	.75	2065	.622	.002400	.001035	2860 30100
16000	15300	do.	"	.83	2290	.615	.002710	.001212	2980 35300
16100	15400	36.0 tons	66500	.97	2580	.604	.003370	.001600	3370 46500 Past Maximum

Failure due to compression.

Thus

$$6 EI y = \frac{P}{2} x^3 - \frac{P}{3} l^2 x$$

$$\int_{\frac{361}{2}}^{\frac{361}{2}} y dx = \frac{1}{6EI} \int_{\frac{361}{2}}^{\frac{361}{2}} \left(\frac{P}{2} x^3 - \frac{P}{3} l^2 x \right) = .00238 \frac{Pl^4}{EI}$$

$$\frac{.00238 Pl^4}{EI} \div .1391 = \frac{.01713 Pl^3}{EI} \text{ average deflection}$$

$$\text{Center deflection} = \frac{23}{1296} \frac{Pl^3}{EI} = .01772 \frac{Pl^3}{EI}$$

$$\therefore \text{Ratio } \frac{\text{Average}}{\text{Center}} = \frac{.01713}{.01772} = .967$$

As the curve for the higher loads after cracks have formed is sharper than that indicated by theory, the coefficient .96 was used. Since for one-third point loading on 12-ft. span $M = 24P$, the equivalent one-third point loading due to thrust and deflection is equal to $\frac{.96 \text{ Center deflection} \times \text{Thrust}}{24}$. In the above, the modification of the form of the deflection curve due to the end thrust was neglected. Table 6 gives the computed experimental values for Beam 453.3. The total equivalent one-third point load having been found, the total moment is obtained by multiplying by 24 and from this the ratio of eccentricity $\frac{e}{h} = \frac{M}{Nh}$.

Comparative Theoretical Values.— Theoretical diagrams (18 to 35) for finding the position of the neutral axis and the stress in the upper fiber, based on Equations (5), (6), (8), (9), (13), (14), and (15) to (20), have been prepared to facilitate the work of obtaining the theoretical stresses. In preparing these, the equations were simplified by substituting values for

p, r, n, and α . For the plain beams the equations for K reduce to

$$K = \frac{1}{12\frac{e}{h}} + \frac{1}{2}$$

$$\frac{N}{bhfc} = 1 - \frac{1}{2K}$$

Beams 451. $p = .441\%$ $\alpha = 0$ $n = 12.5$ $r = .90$

$$\text{Case I. } K = \frac{.1878 + \frac{e}{h}}{1.92\frac{e}{h} + .0401}$$

$$\frac{N}{bhfc} = 1.0551 - \frac{.55}{K}$$

$$\text{Case II. } \frac{e}{h} = \frac{\frac{3}{2} - K - \frac{.1323}{K} + \frac{.1193}{K^2}}{3 + \frac{.3306}{K} - \frac{.297}{K^2}}$$

$$\frac{N}{bhfc} = \frac{1}{2}K + .0551 - \frac{.0496}{K}$$

$$\text{Case III. } \frac{e}{h} = \frac{\frac{4}{3} - K - \frac{.1763}{K} + \frac{.1590}{K^2}}{\frac{8}{3} + \frac{.4408}{K} - \frac{.397}{K^2}}$$

$$\frac{N}{bhfc} = \frac{2}{3}K + .1102 - \frac{.09925}{K}$$

Beams 452. $p = .441\%$ $\alpha = 2$ $n = 12.5$ $r = .90$ $r' = .13$

$$\text{Case I. } K = \frac{\frac{e}{h} + .171}{2.065\frac{e}{h} - .0361}$$

$$\frac{N}{bhfc} = 1.165 - \frac{.557}{K}$$

$$\text{Case II. } \frac{e}{h} = \frac{\frac{3}{2} - K + \frac{.1122}{K} + \frac{.0874}{K^2}}{3 + \frac{.9918}{K} - \frac{.3835}{K^2}}$$

$$\frac{N}{bhfc} = \frac{1}{2}K + .1653 - \frac{.064}{K}$$

$$\text{Case III. } \frac{e}{h} = \frac{\frac{4}{3} - K + \frac{.15}{K} + \frac{.1165}{K^2}}{\frac{8}{3} + \frac{1.322}{K} - \frac{.511}{K^2}}$$

$$\frac{N}{bhfc} = \frac{2}{3}K + .3306 - \frac{.1278}{K}$$

Beams 453. $p = .01$ $\alpha = 0$ $n = 12.5$ $r = .89$

$$\text{Case I. } K = \frac{1.521 + 7.34 \frac{c}{h}}{13.5 \frac{e}{h} + .585}$$

$$\frac{N}{bhfc} = 1.125 - \frac{.611}{K}$$

$$\text{Case II. } \frac{e}{h} = \frac{\frac{3}{2} - K - \frac{.293}{K} + \frac{.261}{K^2}}{3 + \frac{.75}{K} - \frac{.668}{K^2}}$$

$$\frac{N}{bhfc} = \frac{1}{2}K + .125 - \frac{.1113}{K}$$

$$\text{Case III. } \frac{e}{h} = \frac{\frac{4}{3} - K - \frac{.39}{K} + \frac{.348}{K^2}}{\frac{8}{3} + \frac{1}{K} - \frac{.8912}{K^2}}$$

$$\frac{N}{bhfc} = \frac{2}{3}K + .25 - \frac{.2225}{K}$$

Beams 454. $p = .01$ $\alpha = 1$ $n = 12.5$ $r = (1-r') = .88$

$$\text{Case I. } K = \frac{.0955}{\frac{e}{h}} + \frac{1}{2}$$

$$\frac{N}{bhfc} = 1.25 - \frac{.625}{K}$$

$$\text{Case II. } \frac{e}{h} = \frac{\frac{3}{2} - K + \frac{.2157}{K^2}}{3 + \frac{1.5}{K} - \frac{.75}{K^2}}$$

$$\frac{N}{bhfc} = \frac{1}{2}K + .25 - \frac{.125}{K}$$

$$\text{Case III. } \frac{e}{h} = \frac{\frac{4}{3} - K + \frac{.289}{K^2}}{\frac{8}{3} + \frac{2}{K} - \frac{1}{K^2}}$$

$$\frac{N}{bhfc} = \frac{2}{3}K + .50 - \frac{.25}{K}$$

Table 7.

Theoretical Values for Beam 453.2

Total $\frac{1}{2}$ Pt. Load Pounds	Total $M = \frac{e}{h}$ Moment Nh in. lb.	Case I		Case II		N $\frac{bh}{fs}$
		K	fc	fc	fs	
10	240	2.60	325			293
921	22 100	1.30	446			"
1 542	37 000	1.08	526			"
2 352	56 500	.925	630			"
3 373	81 000	.830	764	792	2 670	"
4 405	105 800	.760	896	968	3 650	"
5 436	130 600	.725	1042	1144	6 240	"
6 478	155 500	.700	1162	1325	9 550	"
Case III						
7 520	181 000	.583	1090	1510	12 380	293
8 502	206 000	.565	1222	1685	15 150	"
9 615	231 000	.555	1350	1860	17 900	"
10 668	256 000	.542	1470	2060	21 050	"
9 182	220 500	.685	1530	2002	8 220	587
10 125	243 000	.665	1645	2170	10 600	"
11 188	268 500	.638	1740	2310	13 400	578
12 295	295 000	.623	1880	2520	16 600	587
13 380	321 000	.608	2010	2700	19 900	"
14 505	348 000	.594	2145	2900	23 000	"
15 632	375 000	.580	2280	3110	26 000	"
14 400	346 000	.640	2250	3000	17 200	750
15 257	367 000	.635	2410	3190	18 800	778
16 365	393 000	.620	2510	3380	23 150	767
17 390	422 000	.608	2670	3570	26 300	"
18 680	448 000	.588	2750	3770	30 500	739

Table 8 (a).

Plain Concrete Beams

Beam No.	Equi. Load considered	Pt. Load con- sidered	Thrust lb. per sq. in.	Compression Exper- iment	Compression Theoretical Case I	Tension Exper- iment	Tension Theoretical Case I	
450.1	2316	2316	101	357	434	230*	234	Failure on taking off end load.
450.3	3177	2477	102	348	455	307*	248	Vert. load dropped 500 lb. on cracking in tension.
450.5	5437	4554	306	820	940	402*	339	Beam took 540 lb. additional vertical load after cracking in tension.
450.4	10980	8135	767	1952	1890	762*	360	Beam took 1500 lb. additional load after cracking in tension.

*Modulus of elasticity assumed equal in tension and compression.
Table 8 (b)

Reinforced Concrete Beams

Beam No.	Equi. Load considered	Pt. Load con- sidered	Thrust lb. per sq. in.	Compression Exper- iment	Compression Theoretical Case I	Tension Exper- iment	Tension Theoretical Case I	Compression Exper- iment	Compression Theoretical Case I	Tension Exper- iment	Tension Theoretical Case I	Manner of Failure
451.1	9418	4537	100	600	605 (base)	1020	6300	19100	19100	17350	19700	Tension in steel
451.2	8510	7967	100	1550	1260	1910	36100	46000	44800	14620	19700	do.
451.4	14270	13473	574	2340	2100	3040	34300	40100	34000	4730	64740	do.
451.3	12850	11545	600	2530	1992	2870	35000	33100	28000	33500	35000	Compression*
451.5	17120	11080	800	2020	2080	2720	18600	21200	14750	30500	37400	Compression
452.2	12668	10692	305	1572	1190	1900	33600	39700	37200	17350	19700	Tension in steel
452.5	12004	10694	285	1465	1170	1870	38400	40800	39000	14620	19700	do.
452.1	18450	4382	101	495	599 (base)	790	4680	6460	18100	4730	64740	do.
452.3	17660	15960	882	2320	1940	2910	30500	31800	26400	33500	35000	Compression+
452.4	18427	16717	775	2110	1945	2940	32500	38800	33500	30500	37400	do.
453.1	15663	14449	94	1742	1745	2530	38100	44600	41200	30500	37400	Tension
453.2	15232	14536	96.5	1910	1745	2530	37700	43600	39800	30500	37400	do.
453.3	18680	17590	767	2980	2670	3570	35300	31000	26300	30500	37400	Compression
453.4	20095	18715	750	2940	2680	3670	35200	31200	26900	30500	37400	do.

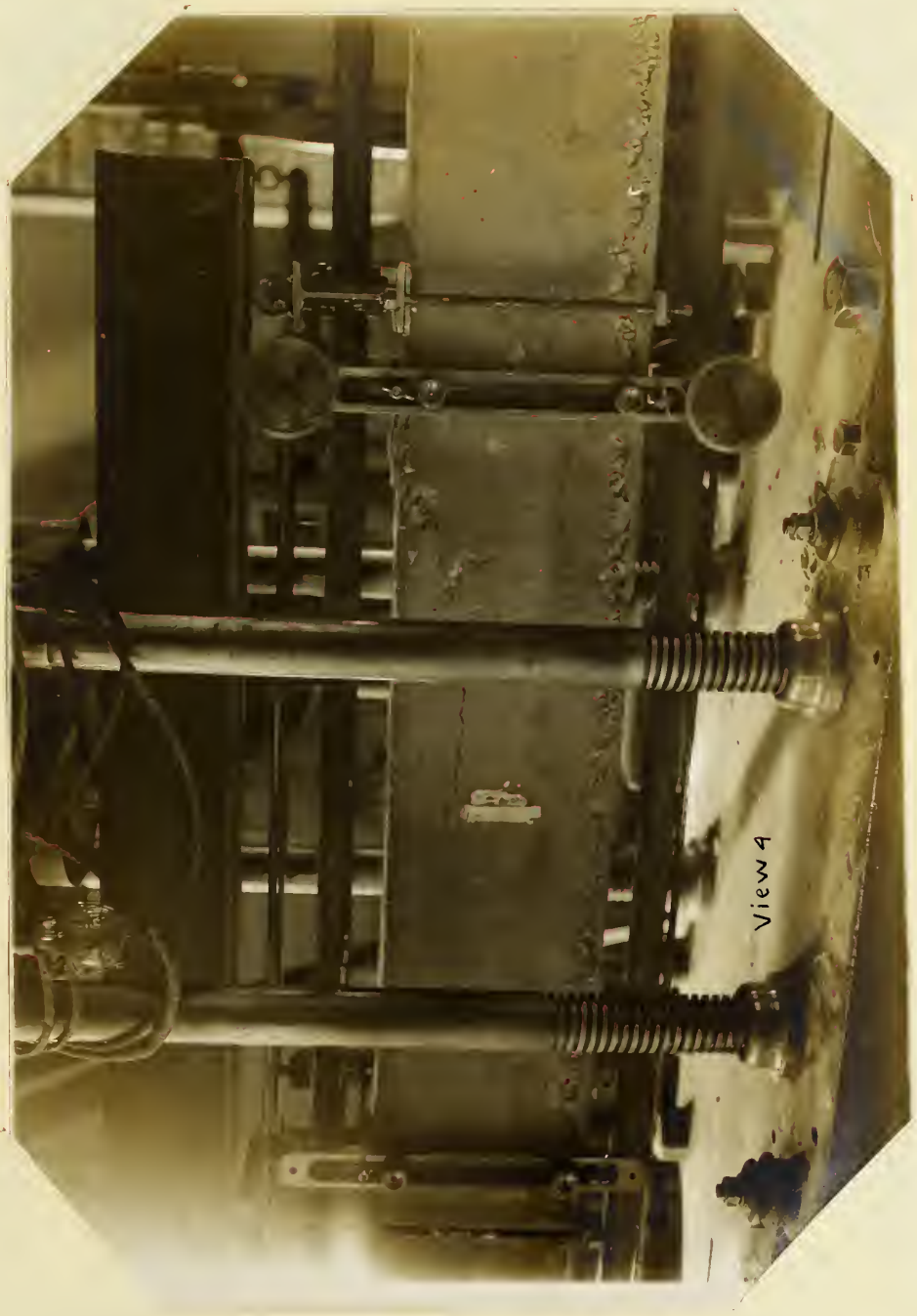
Table 8 (b) continued.

454.1	16565	15530	100	1685	1320	2150	42300	43500	44200	20000	23600	18400	Tension
454.4	16298	15542	105	1805	1365	2220	41900	47300	45700	21600	23500	18100	do.
454.2	24127	22807	918	2810	2340	3580	36700	40200	36400	<i>Past Y.P.</i>	44800	33900	Compression
454.3	24150	23247	752	2530	2215	3390	40800	40750	36600	39600	41200	30900	Tension°

*Thrust at failure = 900 lb. per sq. in.

†Thrust at failure = 805 lb. per sq. in.

°Followed by compression failure.



Having, with the help of these diagrams, obtained K and f_c , f_s , the stress in the steel, is determined by multiplying f_c by the ratio, $\frac{\text{proportionate distance from neutral axis to steel}}{\text{proportionate distance from neutral axis to upper fiber}}$ multiplied by n for Case II and by $2n$ for Case III. This applies to both upper and lower steel. Table 7 gives the computed theoretical values for Beam 453.3.

Plotted Values.— Diagrams 36 to 76 give plots of position of neutral axis, stresses and center deflections against equivalent one-third point loads for the twenty-two beams tested. The intensity of the thrust is also indicated on the diagram. The values on which these diagrams are based are given in the note books, an index to which is given on page 2 of this thesis.

General Summary.— Table 8 gives a summary of loads and stresses for a point at or near the maximum load. View 2 shows the deflected condition of a beam failing by tension in the steel, and View 4 is of a pronounced compression failure.

Manner of Failure.— The plain beams failed in tension by cracking up to the neutral axis at a load that produced a theoretical tensile stress in the lower fiber a little higher than that obtained for the control beams. As noted in Table 8, the beams with high thrust took an increment of vertical load after failing in tension. The actual moment on the section, however, is less than before, since the thrust will produce a negative moment with reference to the center of gravity of the intact section, which counteracts the moment produced by the vertical load. There being no tension present, the total com-

pression on the section will be equal to the thrust, and the remaining external moment will be resisted by this compression acting with a lever arm equal to the distance between the center of the intact section and the center of gravity of the stress acting on it. This lever arm will be one-sixth or one-eighth of the distance from the neutral axis to the upper fiber, according as we assume the stress diagram triangular or parabolic. The following computations were made for two of the plain beams for the point of maximum load:

Beam No.	Vertical Thrust $\frac{1}{2}$ Pt. Load pounds	Neutral Axis Distance lb. from Top	Center Deflection	Lever Arm of Thrust	Equivalent $\frac{1}{2}$ Pt. Load pounds
450.4	8935	73000	6.1 in.	.70 in.	1.93 in. - 5870
450.5	4977	28000	3.22 in.	.76 in.	3.18 in. - 3690

Assuming a parabolic distribution of the compressive stress, its intensity may be calculated by equating external and resisting forces:

Beam No.	Effective $\frac{1}{2}$ Pt. Load	Moment	Compression in Concrete			
			From Thrust	From Moment	Average	From Experiment
450.4	+3065	73600	2230	2940	2585	2940
450.5	+1287	30900	1600	4370	2885	2690

The unit extension of the tension fiber at failure was indicated to be about .0002 under a theoretical stress of about 340 lb. per sq. in. The ratio of eccentricity, $M \div Nh$, at failure in tension was .25, .365, and .74 for thrusts of 770, 306, and 102 lb. per sq. in., respectively, Assuming

the line of pressure at the edge of the middle third, the ratio of eccentricity is .167. From this an indication of the factor of safety in ordinary designs may be obtained.

The reinforced beams behaved in a similar manner at maximum load when failing in tension. They continued taking a vertical load, equal or nearly equal to the maximum, until the deflection became so large that the test could not be further continued. In some cases there are indications that an increment of load was taken after the steel had reached the yield point. The compression failures were more sudden and in a few cases produced complete fracture. For the beams with compressive reinforcement the fracture was not complete, but a large drop in the load was obtained.

Diagonal Tension.— No failure was due to diagonal tension. The highest shear was obtained with Beam 454.3 with the thrust 760 lb. per sq. in., and the shear 120 lb. per sq. in., of section. The neutral axis was at .39 of the height of the beam from the top, and the tension in the steel was 42 000 lb. per sq. in. The highest shear with low thrust obtained with Beam 454.1 was 93 lb. per sq. in. of section, thrust 100 lb. per sq. in. Two diagonal cracks four inches long were formed at seven-eighths of the maximum load but these did not open up farther until failure in tension occurred. These beams had 3/8-in. vertical stirrups, spaced 4 in. centers.

Limits of Error.— The maximum error in determining the thrust would be $1/2$ ton on the gage or 3.75% at 15 tons, and 1.5% at 38 tons. The error in placing the one half inch strips through which the thrust was transmitted, together with the error due to the eccentric distribution of the bearing stress on the same, could not have exceeded one fourth of an inch. With 70 000 pounds thrust this would produce an equivalent one-third point moment load of $\frac{1}{4} \times 70\ 000 \div 24 = 730$ pounds. If to this is added the error due to falling off of load on the machine, the total error in reading the vertical load could not have been over 1000 pounds or 6 $\frac{1}{2}$ % at 16 000 pounds load. The hydraulic jack was not in all cases in line with the axis of the beam, but since the spring dynamometers supporting the thrust system consisting of jack, blocks, and reaction rods, were kept reading the dead weight of the same, it is evident that no error in the vertical or horizontal load could have resulted from this source.

The dimensions of the section and depth to the steel were obtained within one eighth of an inch or 1 $\frac{1}{2}$ % error.

The dials used in measuring deformations were calibrated and found to read accurate within one percent.

In the computed results the upper roller was assumed five and one half inches above the beam and the variation from this could not have been over three sixteenths of an inch. With the neutral axis five inches from the top this gives an error in the computed concrete and steel deformations of 3.75%. The modulus of elasticity of the steel was probably constant within

3 $\frac{1}{2}$ %. The approximations involved in reducing unit deformation in concrete to unit stress, has been discussed elsewhere.

The following comparison of experiment and theory was obtained from the values in Table 8*:

Group I One Half Percent Reinforcement Tension Failures

Beam No.	Tension in Steel			Percent
	Experiment	Case II	Diff.	
451.2	36 100	44 800	-8700	-24.1
451.4	34 300	34 000	+ 300	+ .9
452.2	33 600	37 200	-3600	-10.7
452.5	38 400	39 000	- 600	- 1.6
			Av.	9.3

Group II One Half Percent Reinforcement Compression Failures

Beam No.	Tension in Steel			Percent
	Experiment	Case III	Diff.	
452.4	32 500	38 800	-6300	-19.40
451.3	35 000	33 100	+1900	- 5.42
451.5	18 600	21 200	-2600	+14.00
452.3	30 500	31 800	+1300	- 4.26
			Av.	10.57

Group III One Percent Reinforcement Tension Failures

Beam No.	Tension in Steel			Percent
	Experiment	Case II	Diff.	
453.1	38 100	41 200	-2900	- 7.62
453.2	37 700	39 800	- 2100	- 5.56
454.1	42 300	44 200	- 1900	- 4.50
454.4	41 900	45 700	- 3800	- 9.05
			Av.	6.68

*Beams 451.1 and 451.2 are excluded in this comparison as they were subjected to repetitive loading.

Group IV One Percent Reinforcement Compression Failures

Beam No.	Tension in Steel			
	Experiment	Case III	Diff.	Percent
453.3	35 300	31 000	+ 4300	+ 12.2
453.4	35 200	31 200	+ 4000	+ 11.35
454.2	36 700	40 200	- 3500	- 9.45
454.3	40 800	40 750	+ 50	+ .01
			Av.	<u>8.00</u>

For the beams in Group I for a point near the maximum, a variation in the vertical load of 1000 pounds would produce a change of stress in the steel of 7500 lb. per sq. in. The average maximum one third point load on these beams was 11 860 lb. Similarly, for the beams in Group II a variation of the vertical load of 1000 pounds would change the tension in the steel by about 5000 lb. per sq. in. The average maximum one third point load on these beams was 15 880 pounds. For Group III a variation in the moment load of 1000 pounds corresponds to 3300 lb. per sq. in. change of stress in the steel, the average maximum vertical load being 15 920 pounds. For Group IV a similar variation in the vertical load will change the stress in the steel about 4000 lb. per sq. in., the average maximum moment load being 22 000 pounds.

Comparing the above with the differences between experimental and theoretical values given for the several groups, it is seen that, but for one exception in Group I, experiment agrees with theory within the limits of accuracy obtained for

the loads, plus errors involved in the reduction of experimental data.

CONCLUSIONS

From the data and the foregoing discussion the following conclusions may be drawn.

A plain concrete member designed with the line of pressure at the edge of the middle third will have a factor of safety of two against failure in tension, for thrusts not exceeding three hundred pounds per square inch for the entire section. Failure in tension does not imply failure of the member as a whole if suitable provisions are made for sustaining the reaction of the thrust. There are indications that the modulus of elasticity for the tension side is considerably lower than that for the compression side for the same unit deformation.

For reinforced members under working loads, straight line variation of compressive stress, neglecting tension in the concrete, should be used for obtaining the compressive stress in the concrete and the tensile stress in the steel, as well as the stress in the compression steel in case of double reinforcement.

For loads developing the ultimate strength of the concrete in compression, the compressive stress in the concrete and tensile stress in the steel will be safely given by adding to the values obtained by considering parabolic variation of the compressive stress, one fourth of the difference between the same and the values obtained by considering straight line variation for the same load. For the compression steel one

fourth of the difference between the corresponding values should be subtracted from the value obtained ^{by} considering parabolic variation.

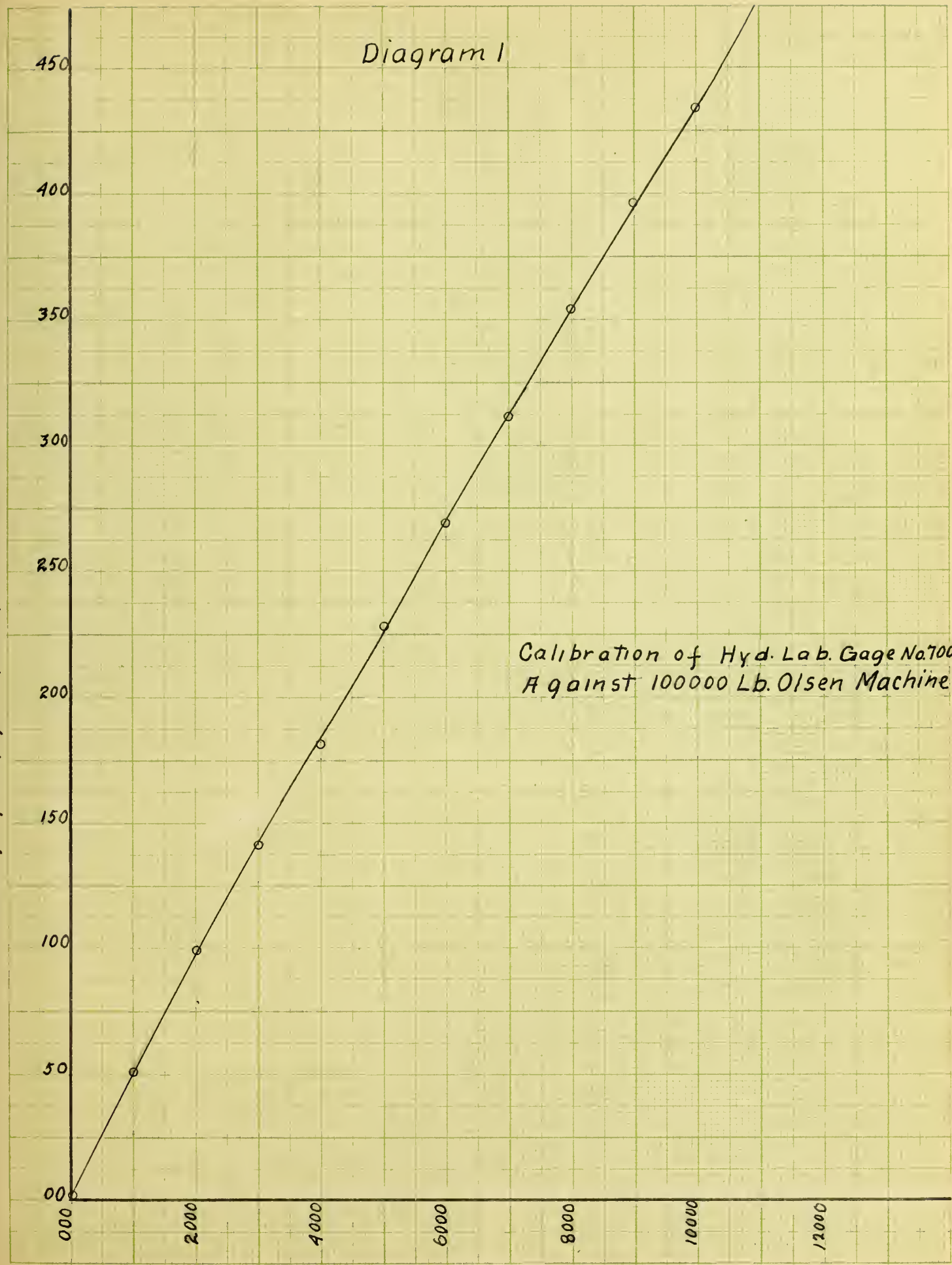
In general, we are led to conclude that the formulae for Flexure and Direct Stress as they apply to concrete and reinforced concrete will give values for the stresses in the materials of a member subject to axial load and bending well within the limit of accuracy required in design.

STRESS-LOAD DIAGRAMS.

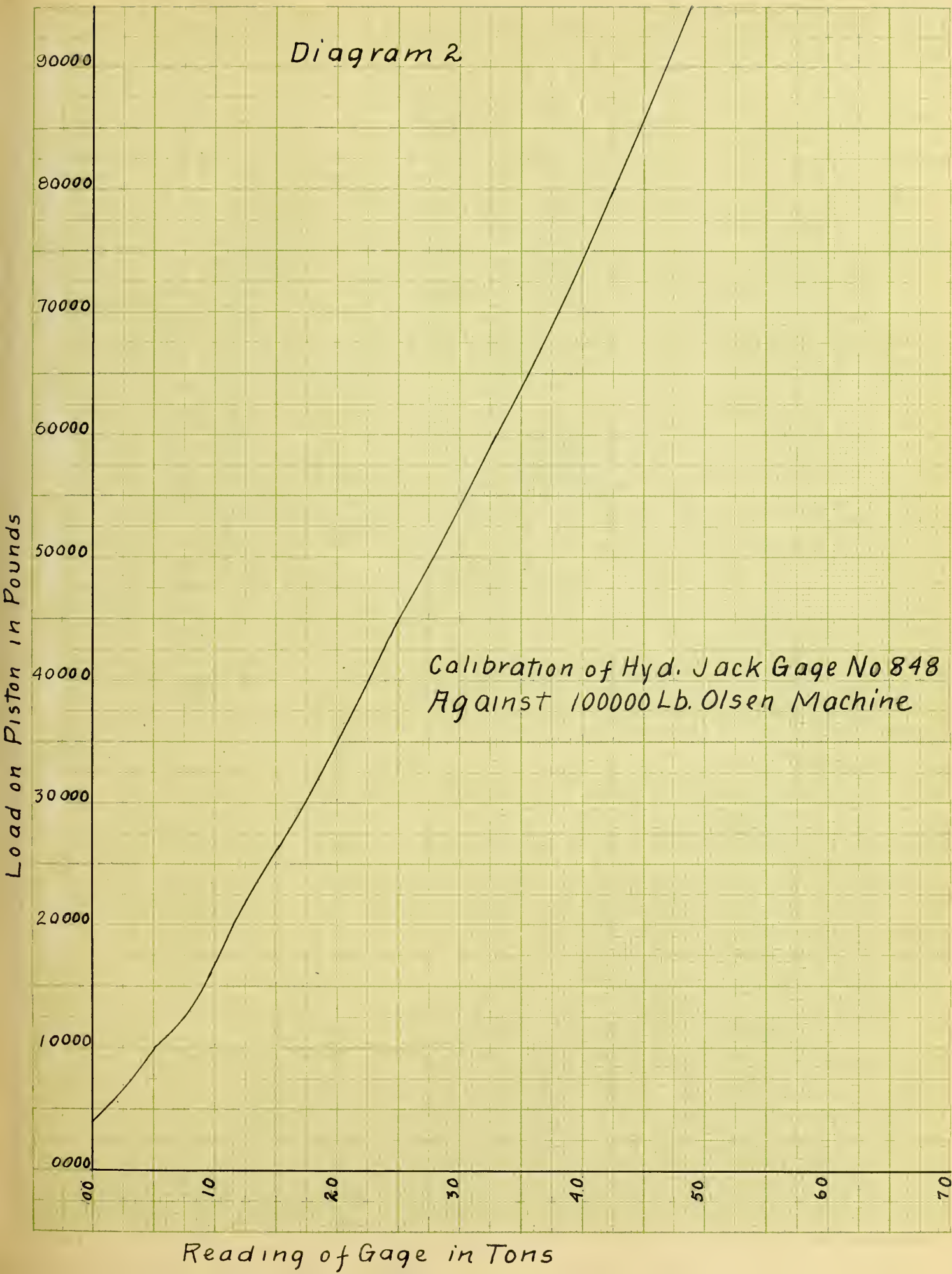
Reading of Gage, Lb. per sq. in. Pressure

Diagram 1

Calibration of Hyd. Lab. Gage No. 700
Against 100000 Lb. Olsen Machine



Load on Piston in Pounds



Modulus of Elasticity in Million Lb. per sq. in.

55

Diagram 3

Average Modulus
of

Ten Test Cylinders

450.2 452.3

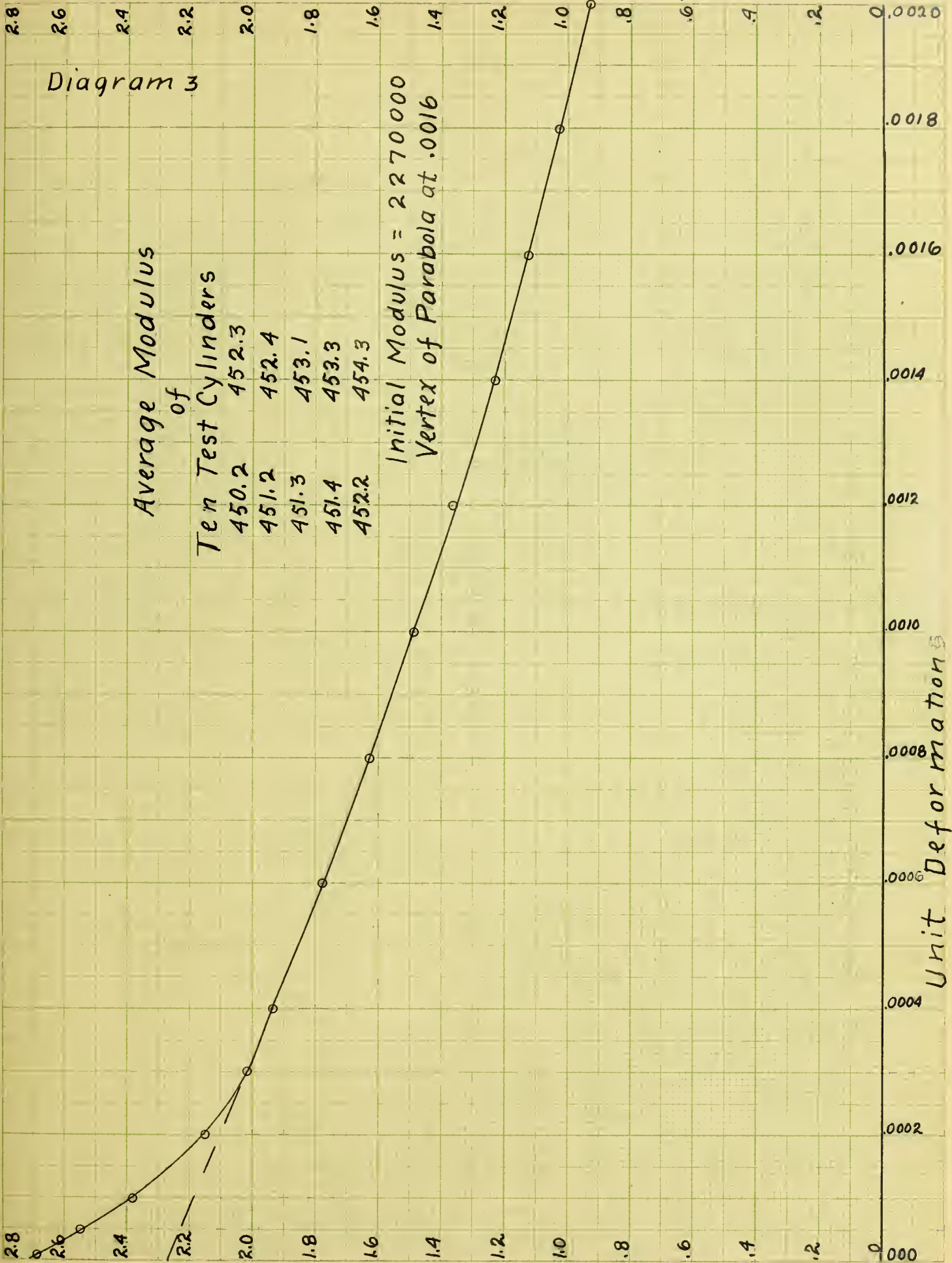
451.2 452.4

451.3 453.1

451.4 453.3

452.2 454.3

Initial Modulus = 2270000
Vertex of Parabola at .0016



Modulus of Elasticity in Million Lb. per sq. in.

52

Diagram 4

Modified Curve For
Modulus of Elasticity of
Concrete in Compression.
Vertex of Parabola at .0026

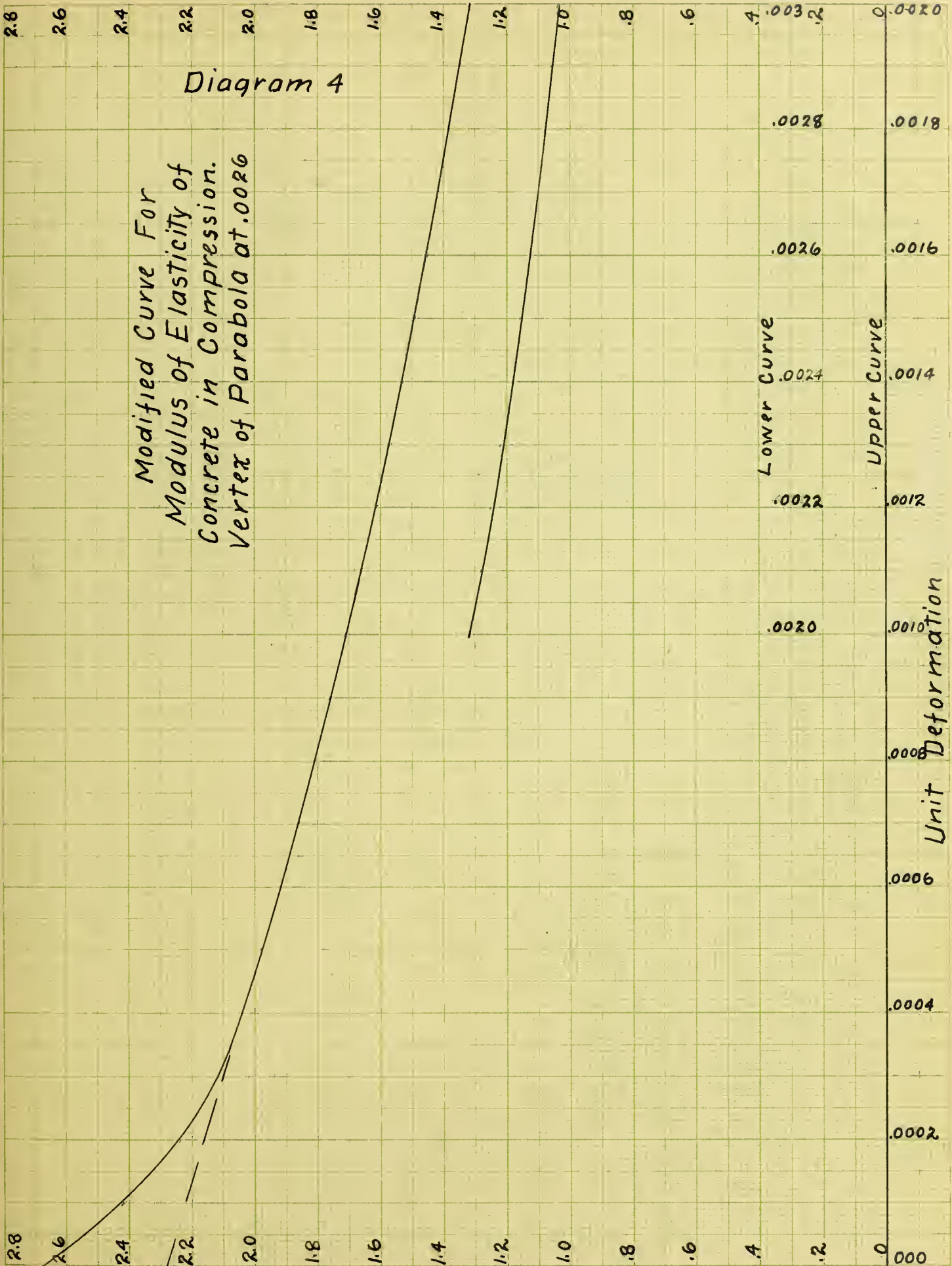
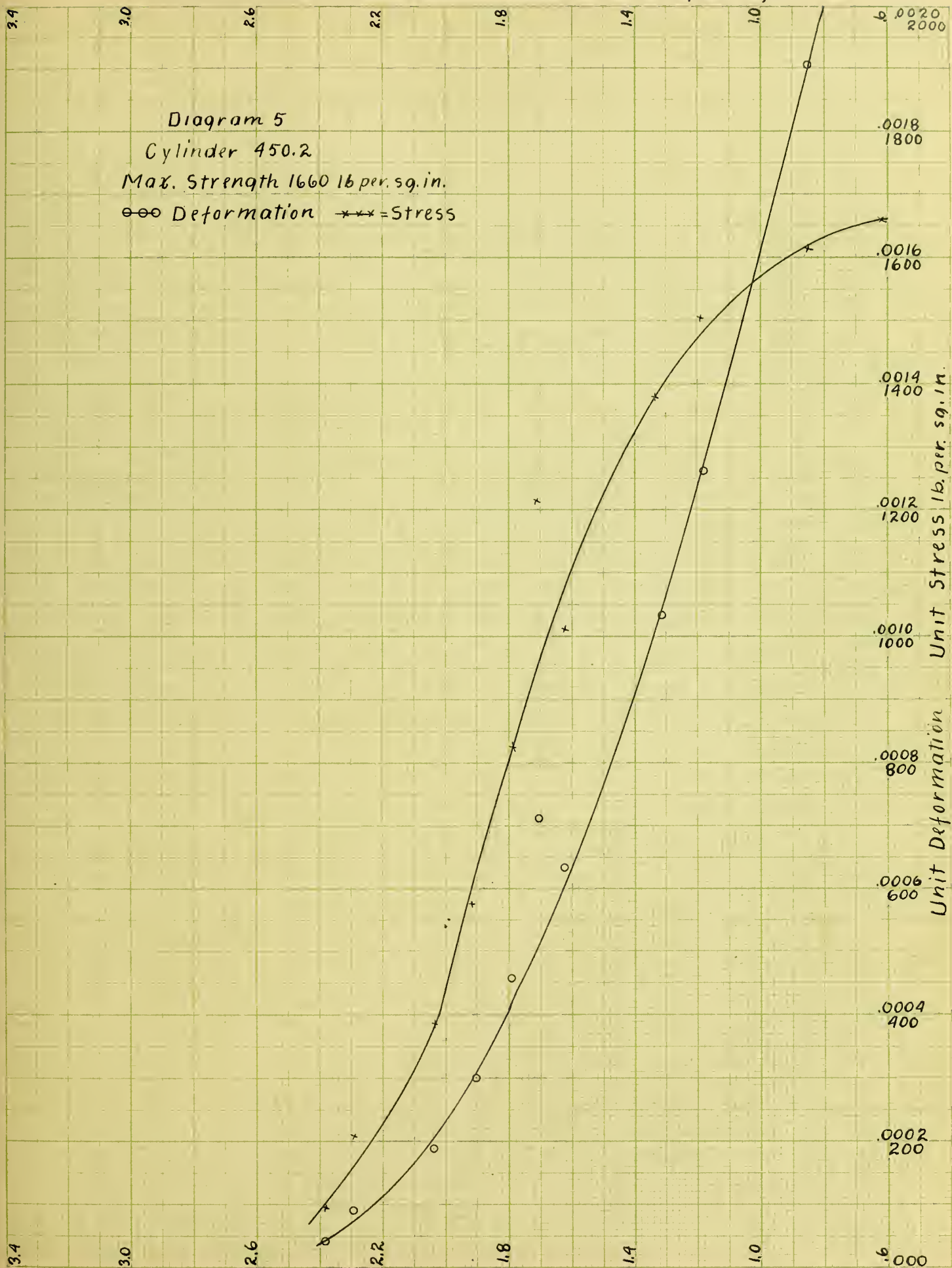


Diagram 5

Cylinder 450.2

Max. Strength 1660 lb per. sq. in.

ooo Deformation *** = Stress



Modulus of Elasticity in Million Lb. per sq. in.

58

3.4

3.0

2.6

2.2

1.8

1.4

1.0

.0020

.0020

2000

Diagram 6

Cylinder No. 451.2

Max. strength 2405 lb per sq. in.

--- Deformation --- Stress

.0018
1800

.0016
1600

.0014
1400

.0012
1200

.0010
1000

.0008
800

.0006
600

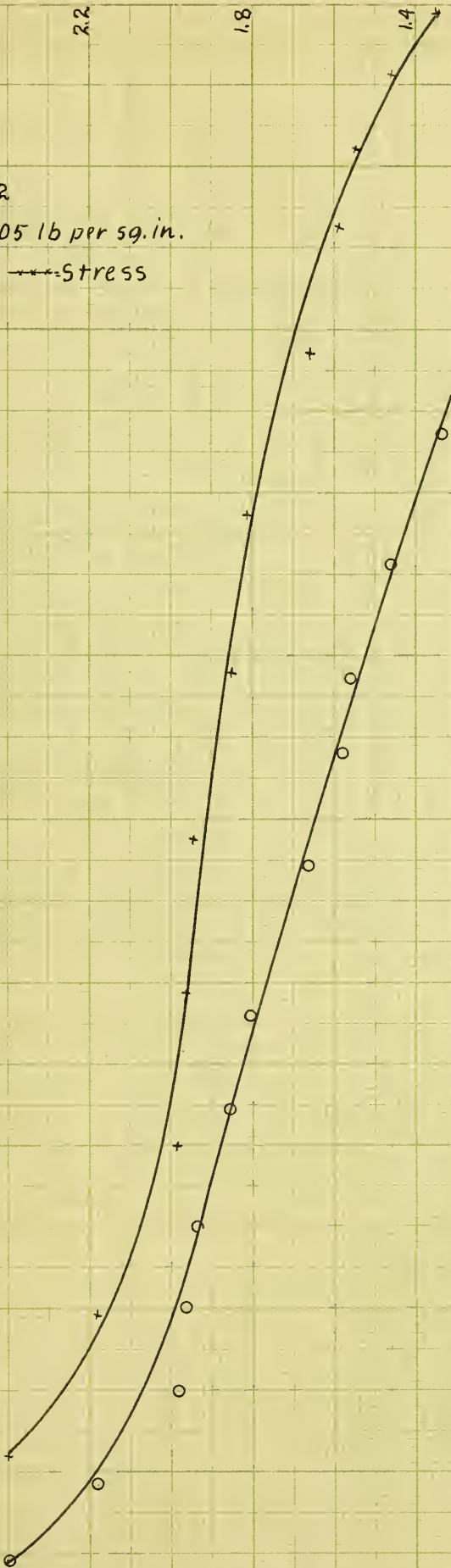
.0004
400

.0002
200

000

Unit Stress lb. per sq. in.

Unit Deformation



Modulus of Elasticity in Million Lb per sq.in.

59

.0020
2000

Diagram 7

Cylinder No 451.3

Max. Strength 1650 lb. per sq.in.

ooo Deformation *** Stress

.0018
1800

.0016
1600

.0014
1400

.0012
1200

.0010
1000

.0008
800

.0006
600

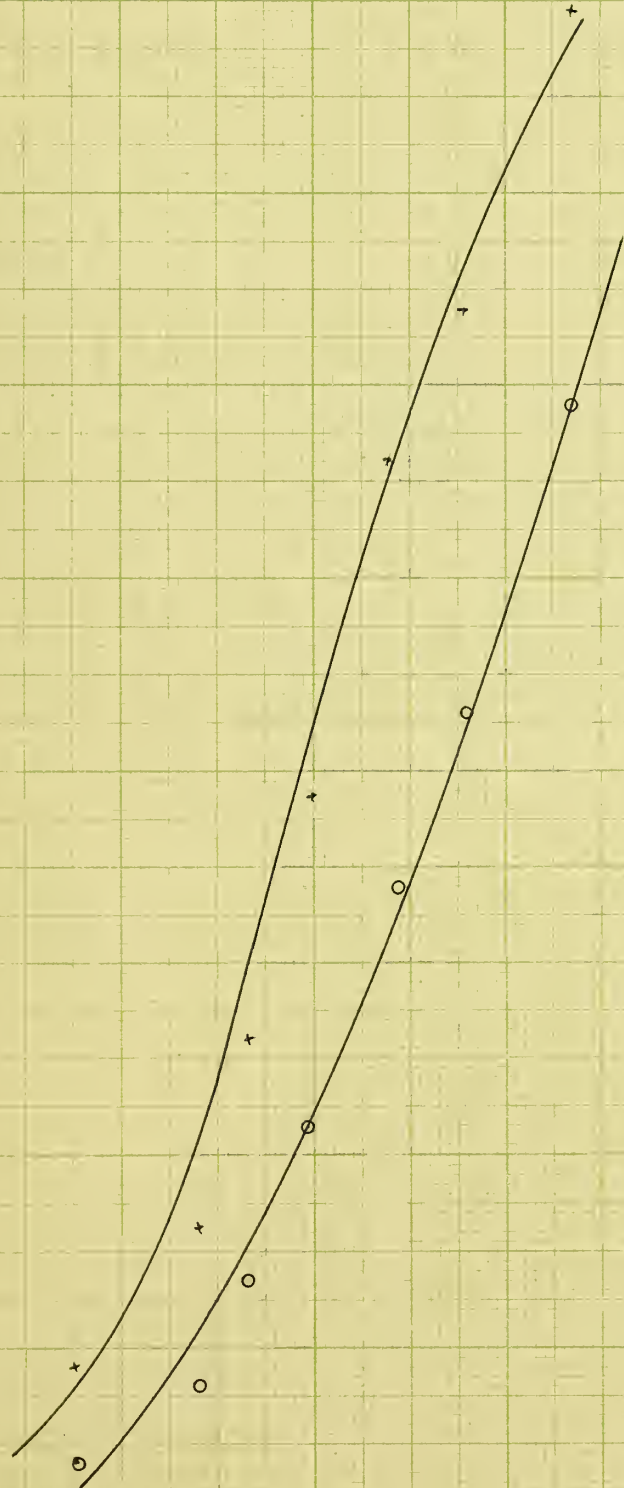
.0004
400

.0002
200

000

Unit stress lb per sq.in

Unit Deformation



Modulus of Elasticity in Million Lb. per sq. in.

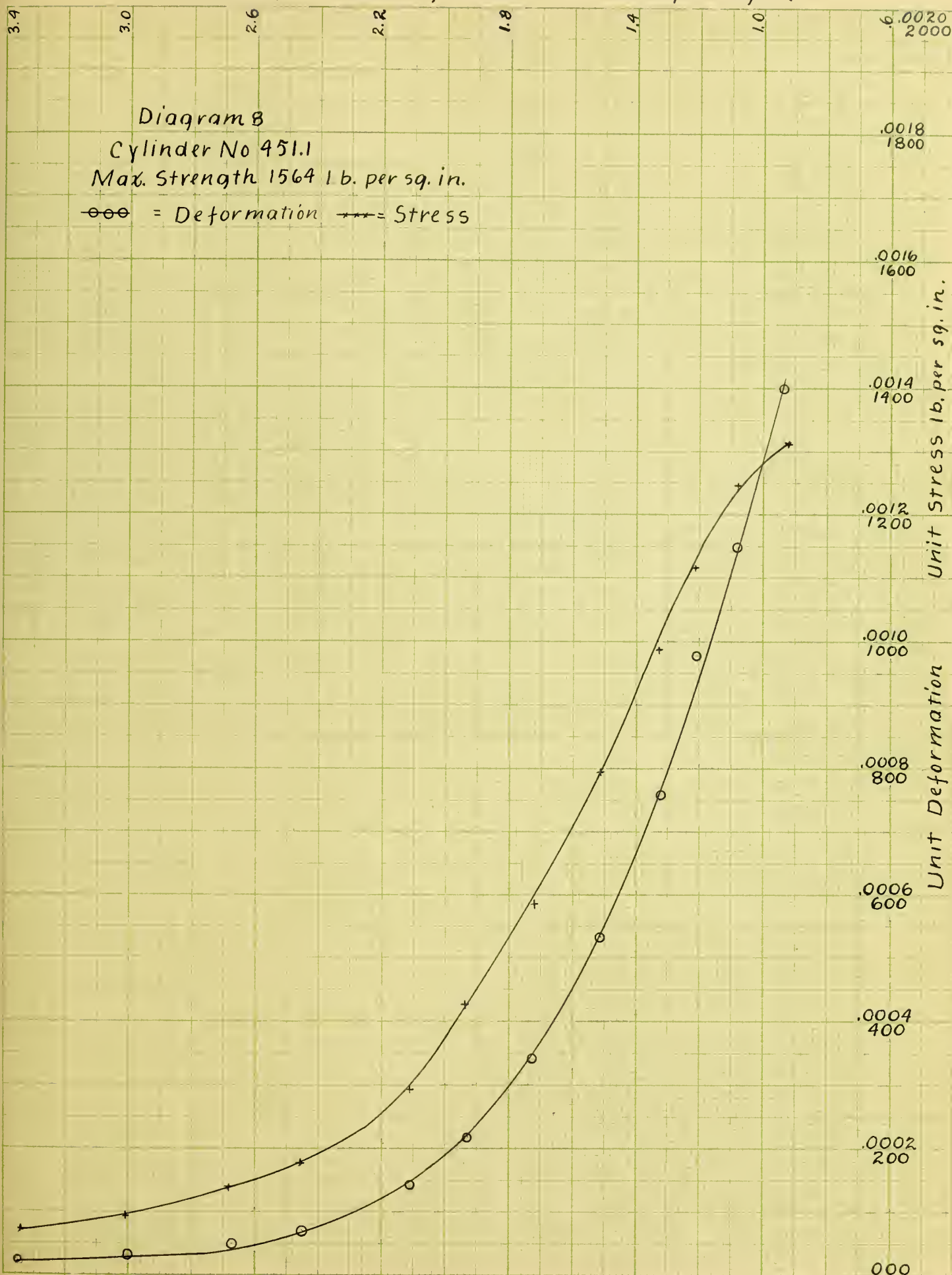
60

Diagram 8

Cylinder No 451.1

Max. Strength 1564 lb. per sq. in.

ooo = Deformation +++ = Stress



Modulus of Elasticity in Million L.b. per sq. in.

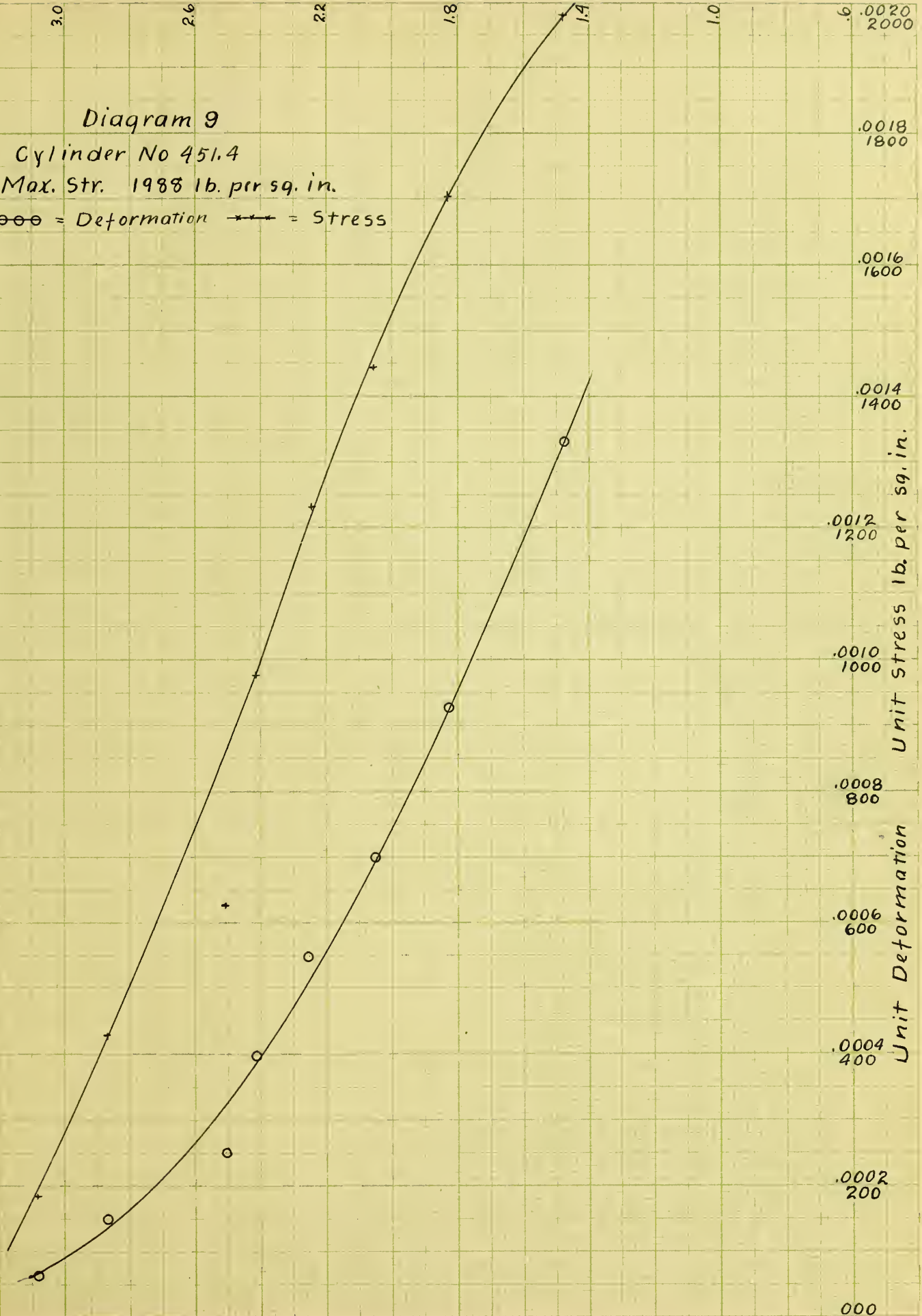
61

Diagram 9

Cylinder No 451.4

Max. Str. 1988 lb. per sq. in.

ooo = Deformation +--+ = Stress



Modulus of Elasticity in Million Lb. per sq. in.

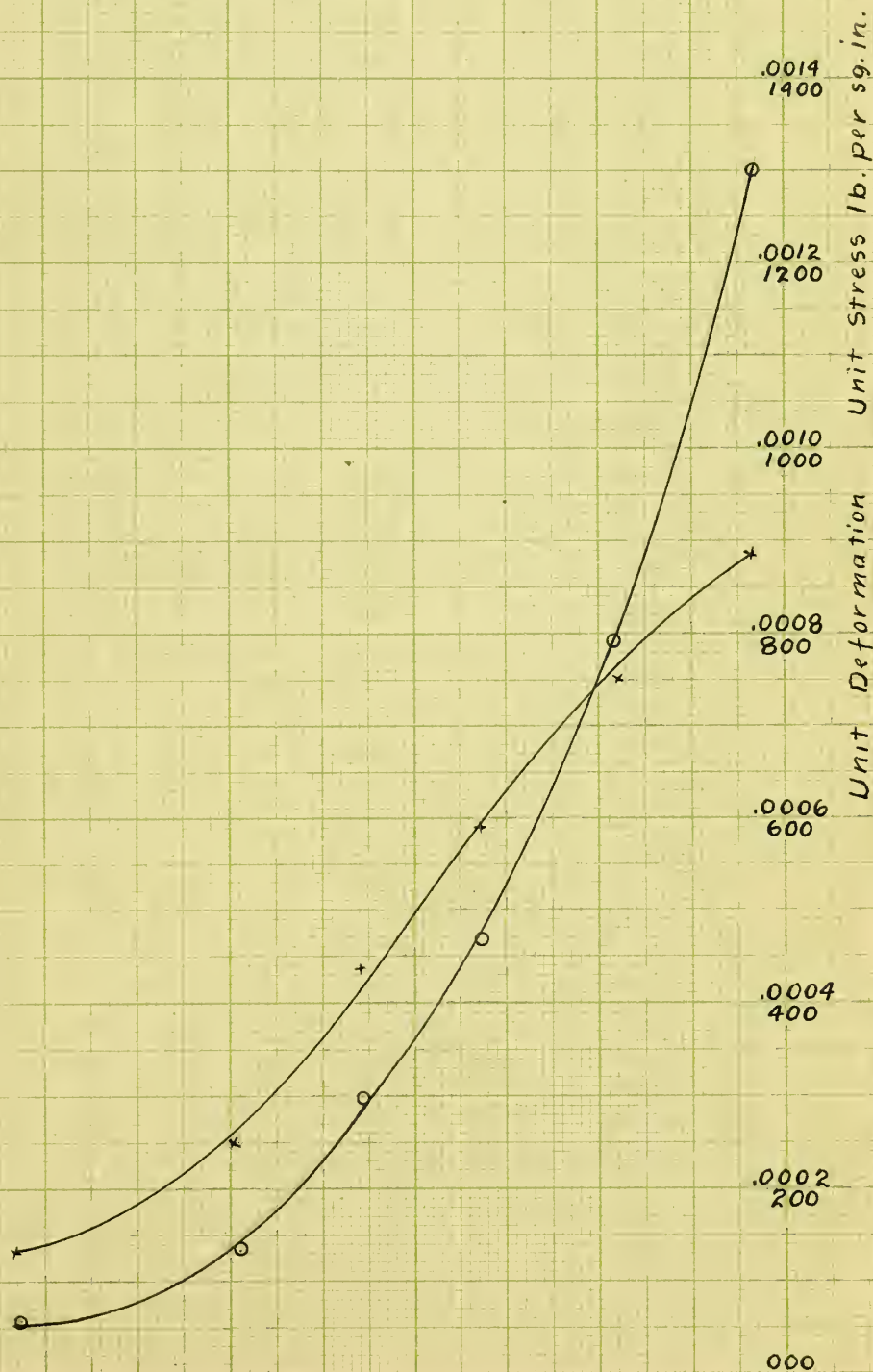
62

Diagram 10

Cylinder No. 452.1

Max. Strength 1150 lb. per sq. in.

---○--- = Deformation ---+--- = Stress



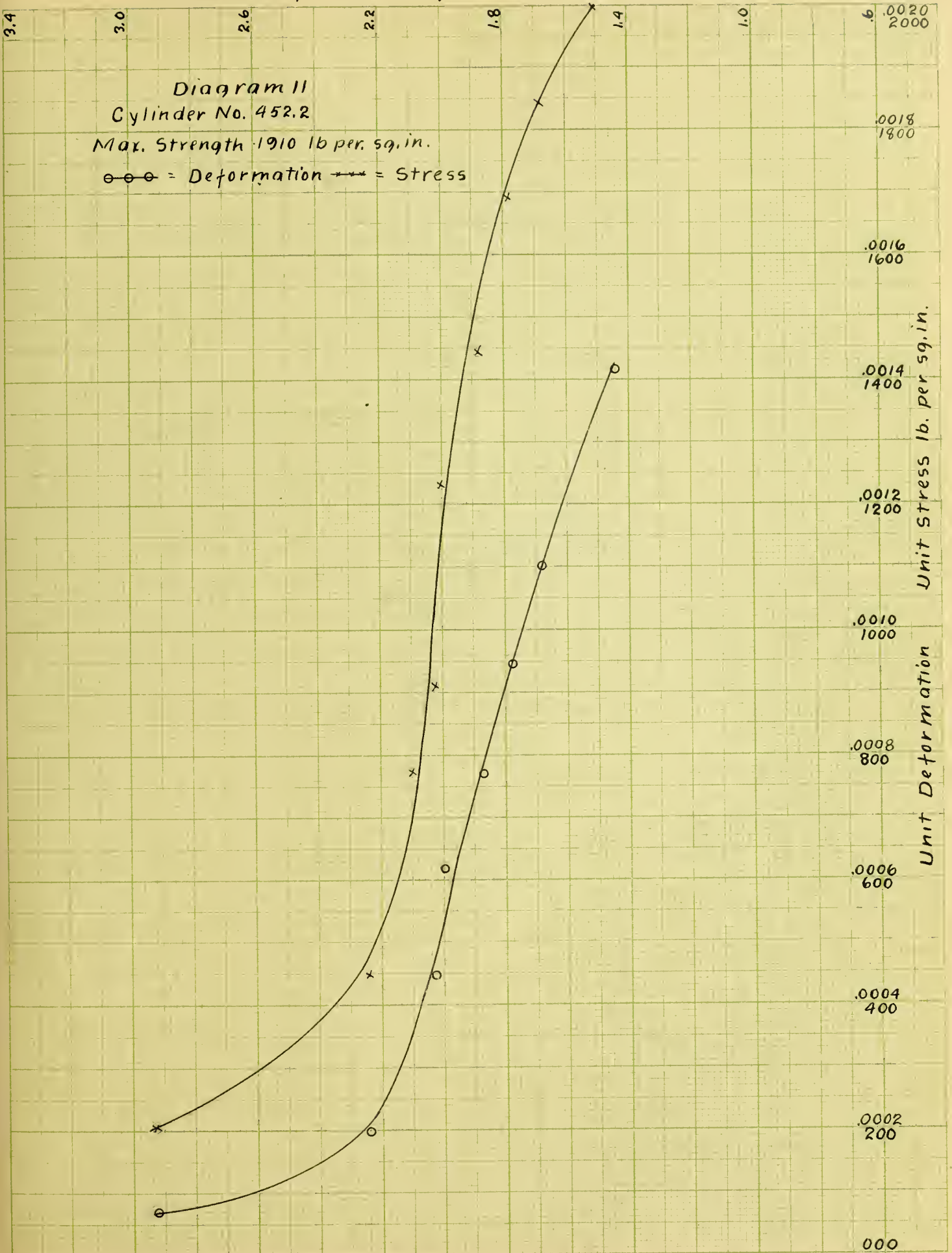
Modulus of Elasticity in Million Lb. per sq. in.

63

Diagram II
Cylinder No. 452.2

Max. Strength 1910 lb per. sq. in.

○-○-○ = Deformation +--+ = Stress



3.4

3.0

2.6

2.2

1.8

1.4

1.0

Diagram 12
Cylinder No. 452.3

Maximum Strength 1910 lb. per sq. in.

○—○ = Deformation —+— = Stress

.0018
1800

.0016
1600

.0014
1400

.0012
1200

.0010
1000

.0008
800

.0006
600

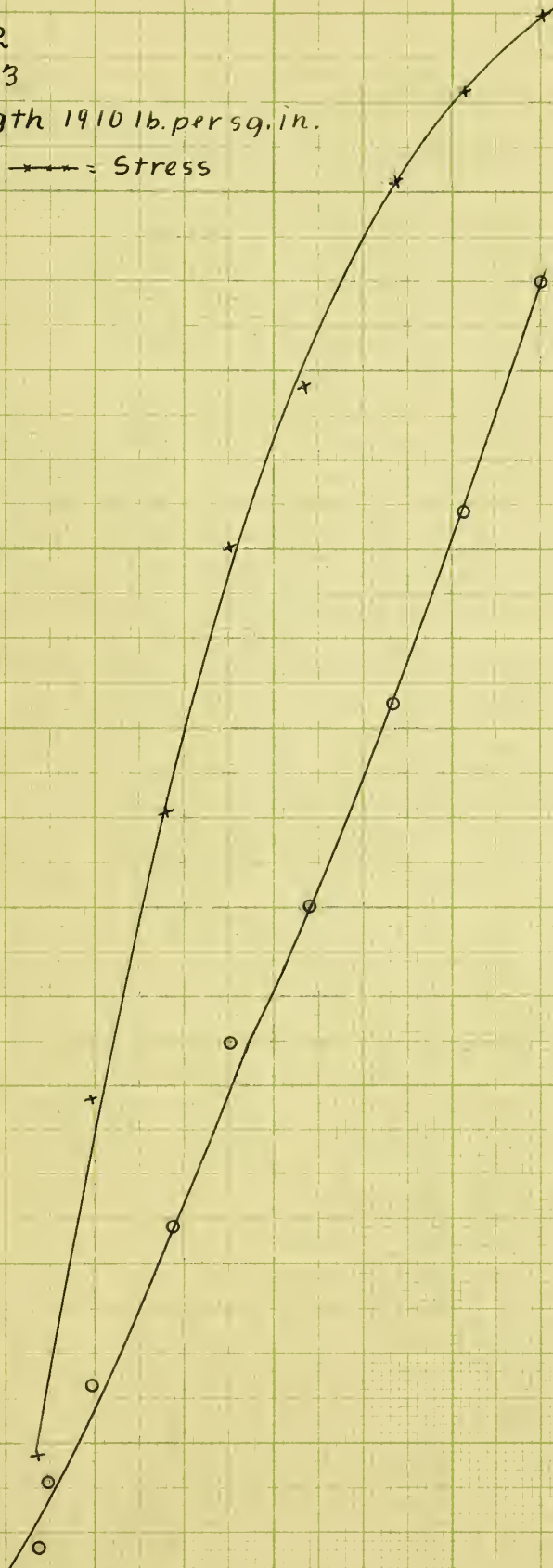
.0004
400

.0002
200

000

Unit stress lb. per sq. in.

Unit Deformation



3.4

3.0

2.6

2.2

1.8

1.4

1.0

.0020
2000

Diagram 13

Cylinder No. 452.4

Max. Strength 1890 lb. per sq. in.

○ ○ ○ = Deformation × × × = Stress

.0018
1800

.0016
1600

.0014
1400

.0012
1200

.0010
1000

.0008
800

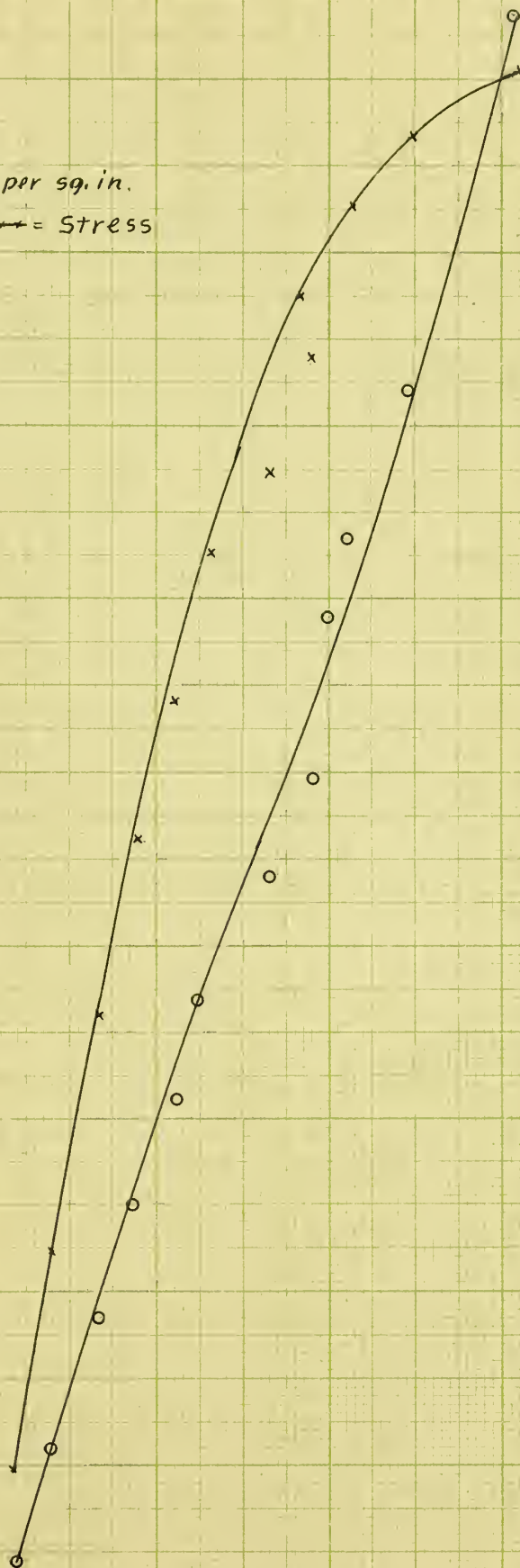
.0006
600

.0004
400

.0002
200

000

Unit Stress lb. per sq. in.
Unit Deformation

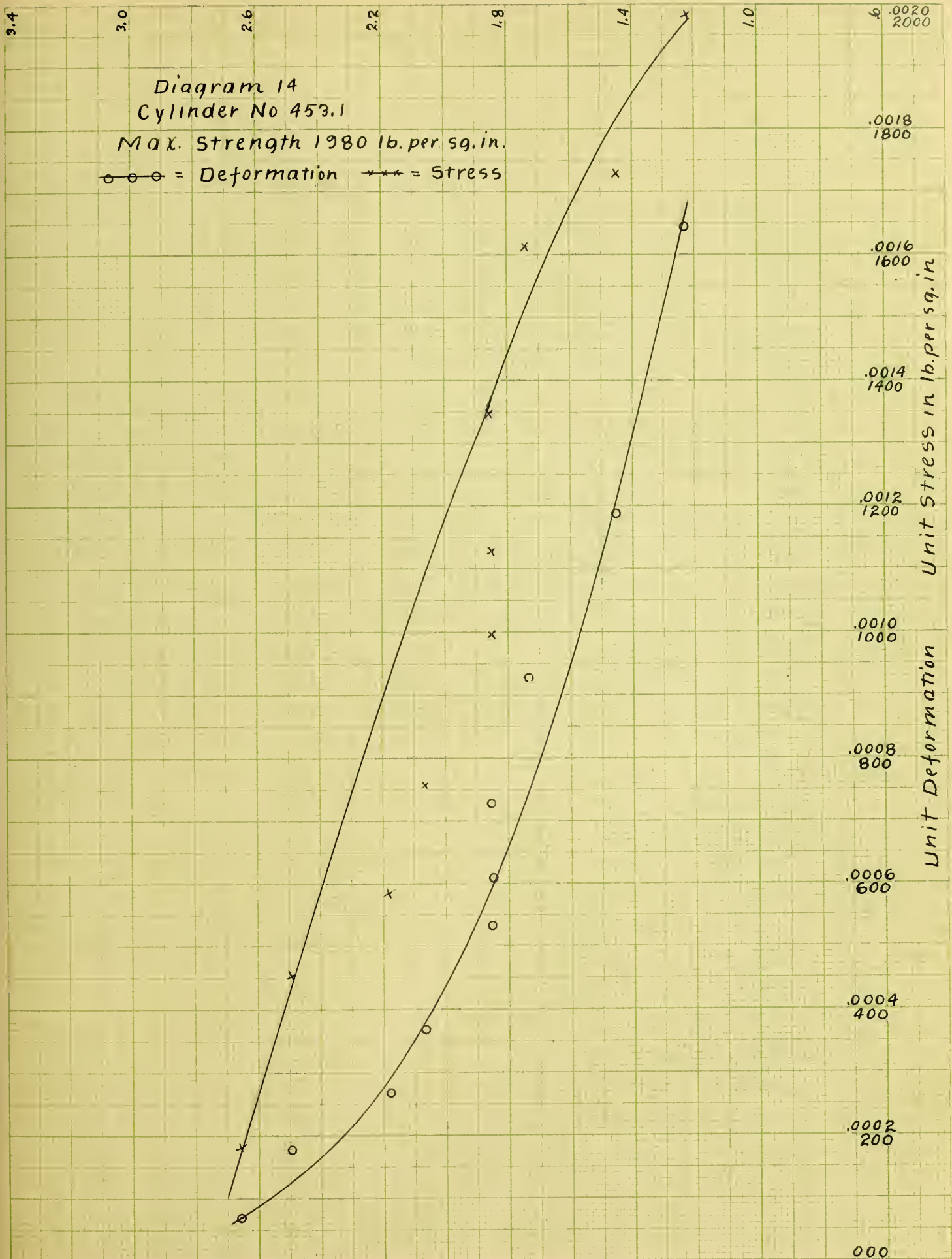


Modulus of Elasticity in Million L b. per sq. in.

Diagram 14
Cylinder No 453.1

Max. Strength 1980 lb. per sq. in.

○ ○ ○ = Deformation × × × = Stress



3.4

3.0

2.6

2.2

1.8

1.4

1.0

0.9

0.8

0.7

0.6

Diagram 15

Cylinder No. 453.3

Max. Strength 1562 lb. per sq. in.

○ ○ ○ = Deformation × × × = Stress

.0020
2000

.0018
1800

.0016
1600

.0014
1400

.0012
1200

.0010
1000

.0008
800

.0006
600

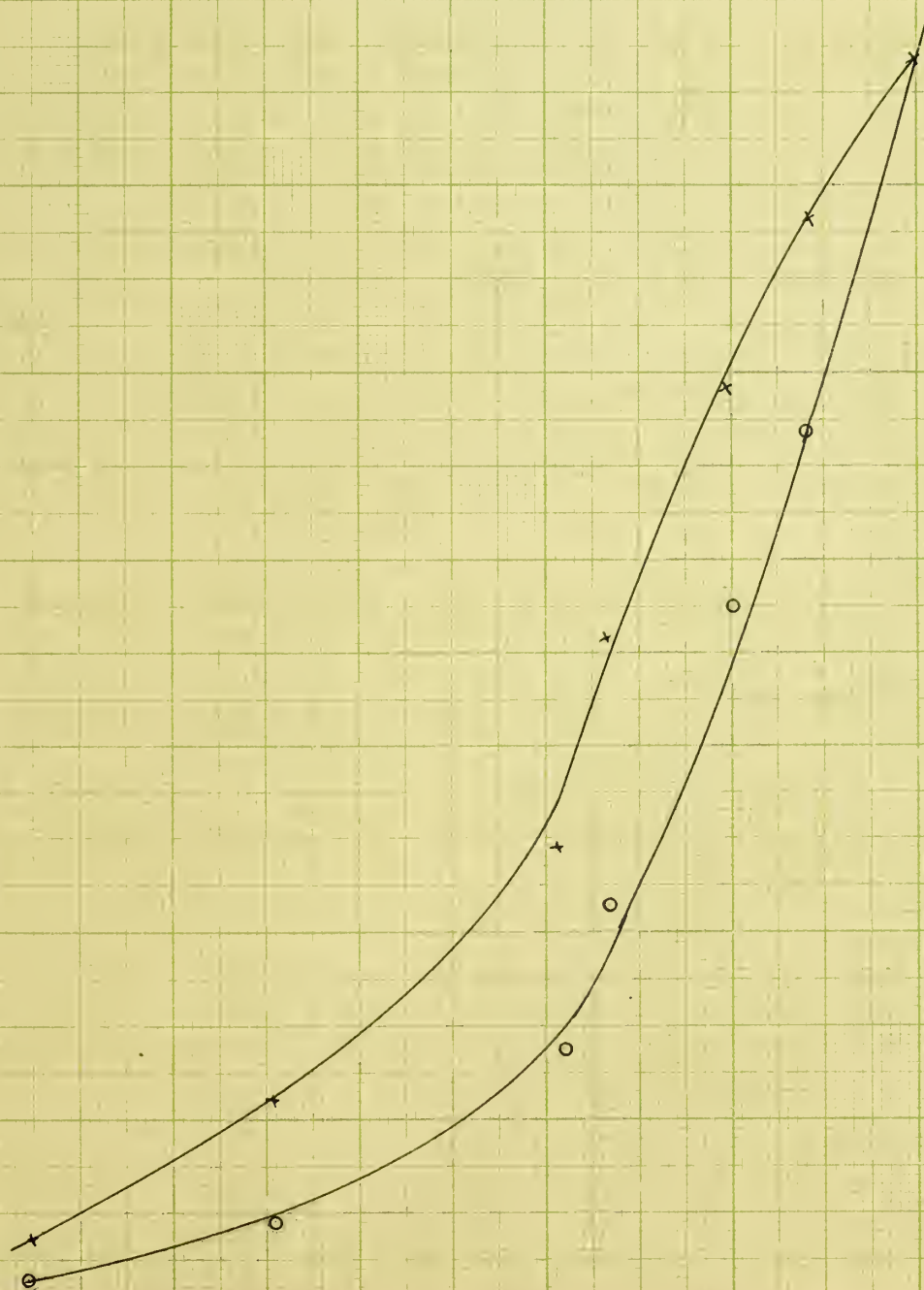
.0004
400

.0002
200

000

Unit Stress lb. per sq. in.

Unit Deformation



3.4

3.0

2.6

2.2

1.8

1.4

1.0

.0020
2000

Diagram 16

Cylinder No. 454.1

Max. Strength 1057 lb per sq. in.

---○--- = Deformation ---x--- = Stress

.0018
1800

.0016
1600

.0014
1400

.0012
1200

.0010
1000

.0008
800

.0006
600

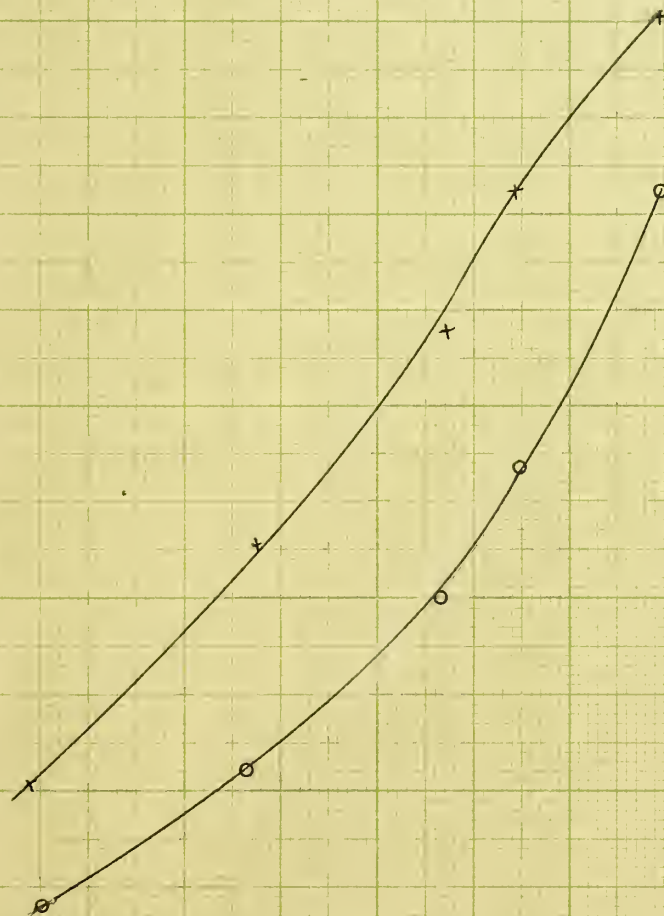
.0004
400

.0002
200

000

Unit Stress lb. per sq. in.

Unit Deformation



Modulus of Elasticity in Million Lb. per sq. in.

69

3.4

3.0

2.6

2.2

1.8

1.4

1.0

.6

.0020

2000

Diagram 17

Cylinder No. 454.3

Max. Strength 1779 lb. per sq. in

○-○-○ = Deformation x-x-x = Stress

.0018
1800

.0016
1600

.0014
1400

.0012
1200

.0010
1000

.0008
800

.0006
600

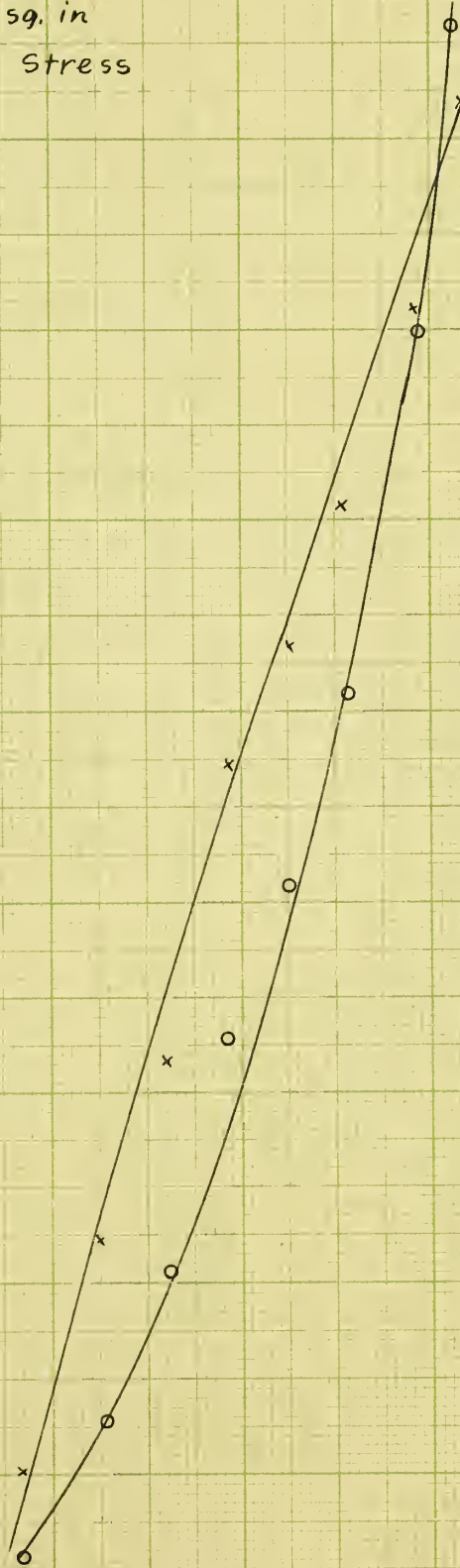
.0004
400

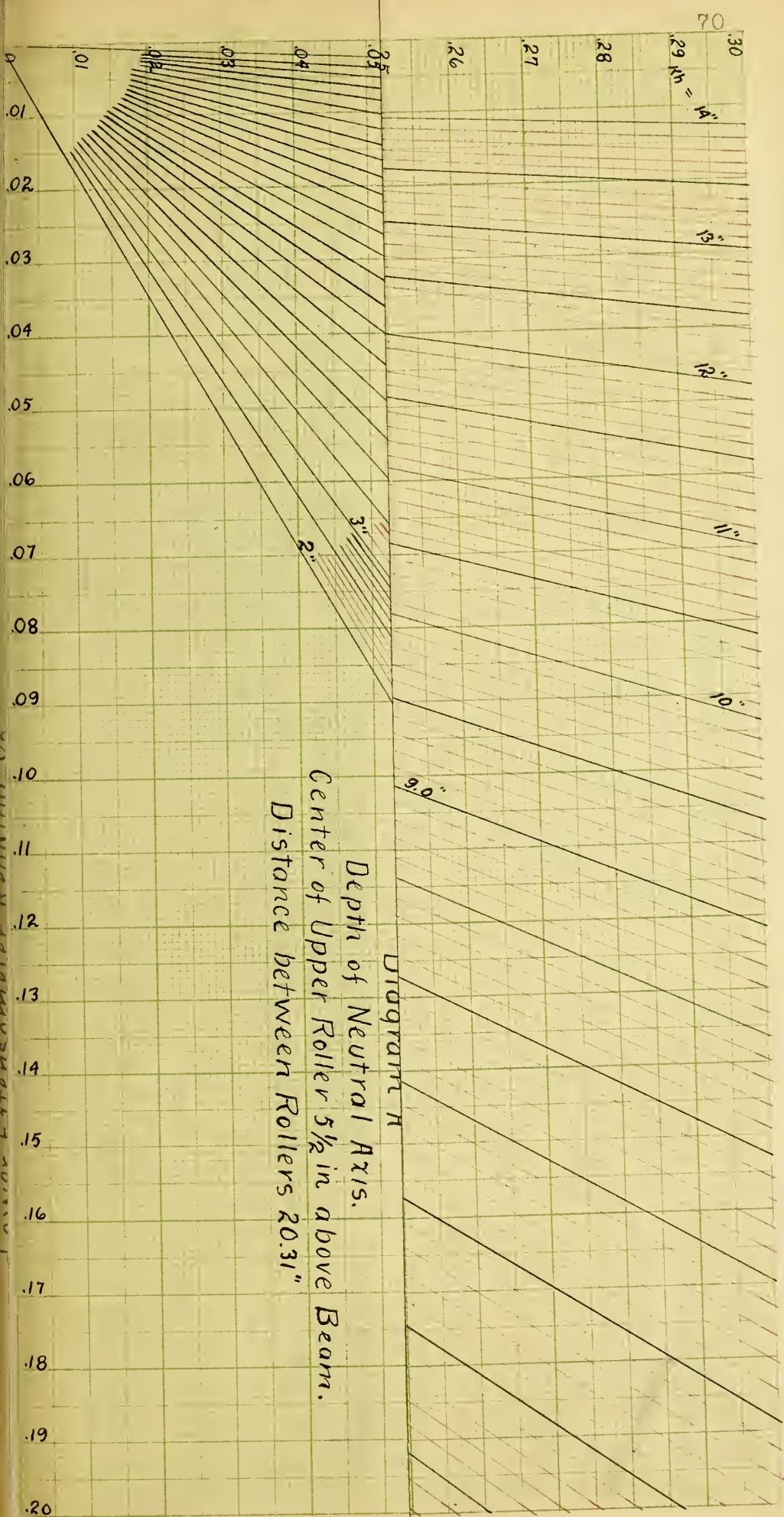
.0002
200

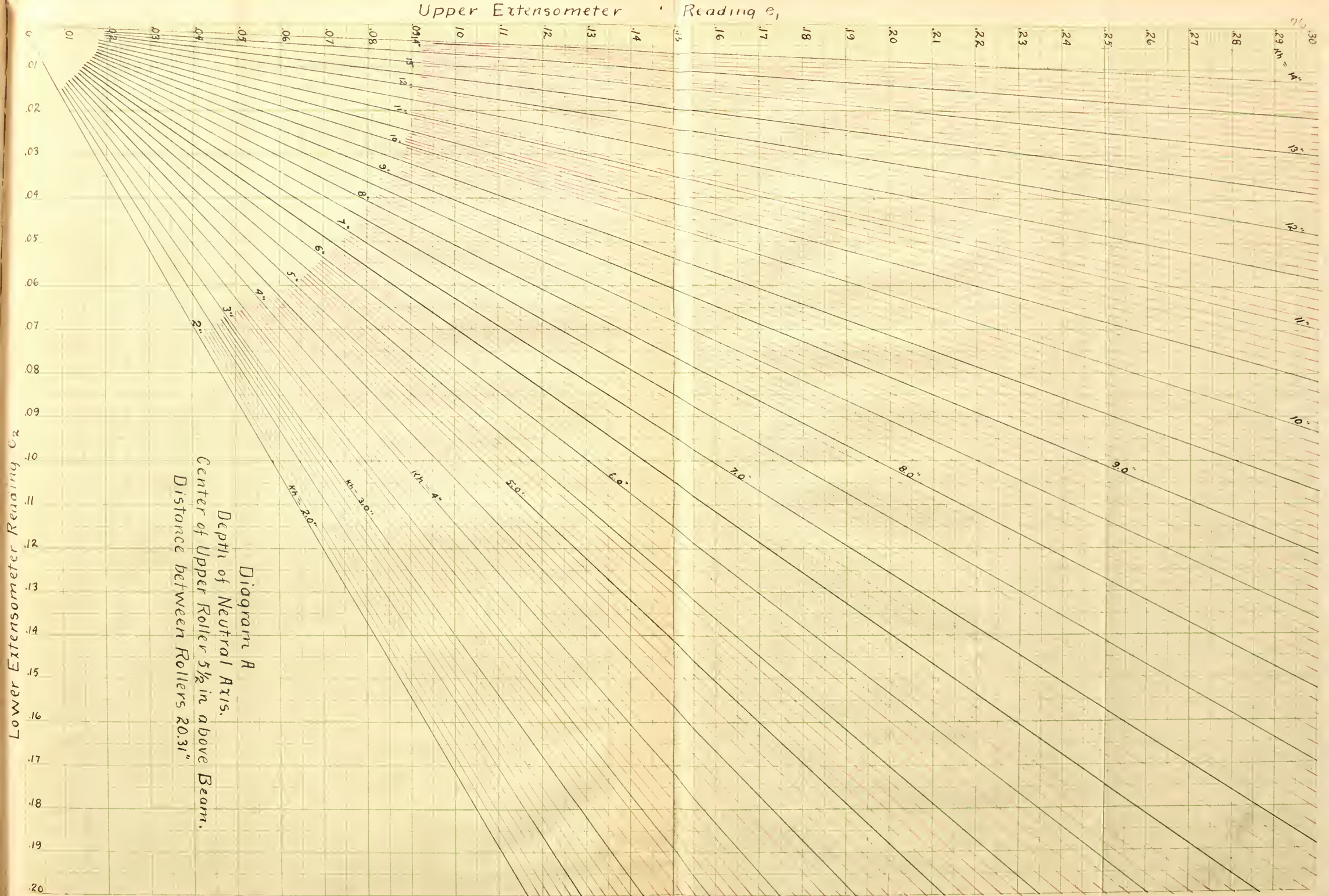
000

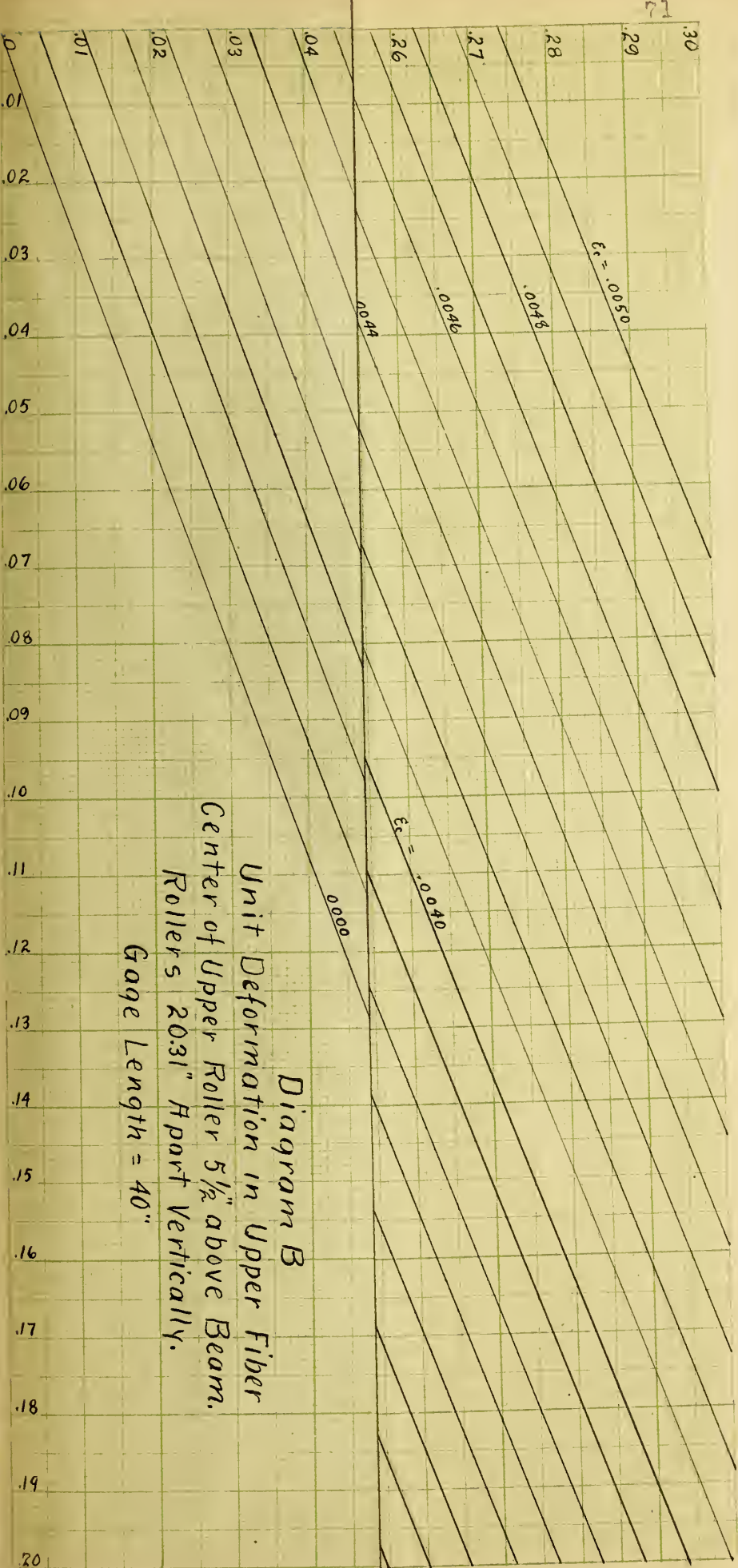
Unit Stress lb. per sq. in.

Unit Deformation









30

29

0500. = 3

.0048

0046

0044

.0042

$$\epsilon_c = +0.0040$$

~~0038~~

.6036

.003

.003

00.

00.

.002

002

.002

007

100.

1001

100.

100.

1001

500

000

50

100

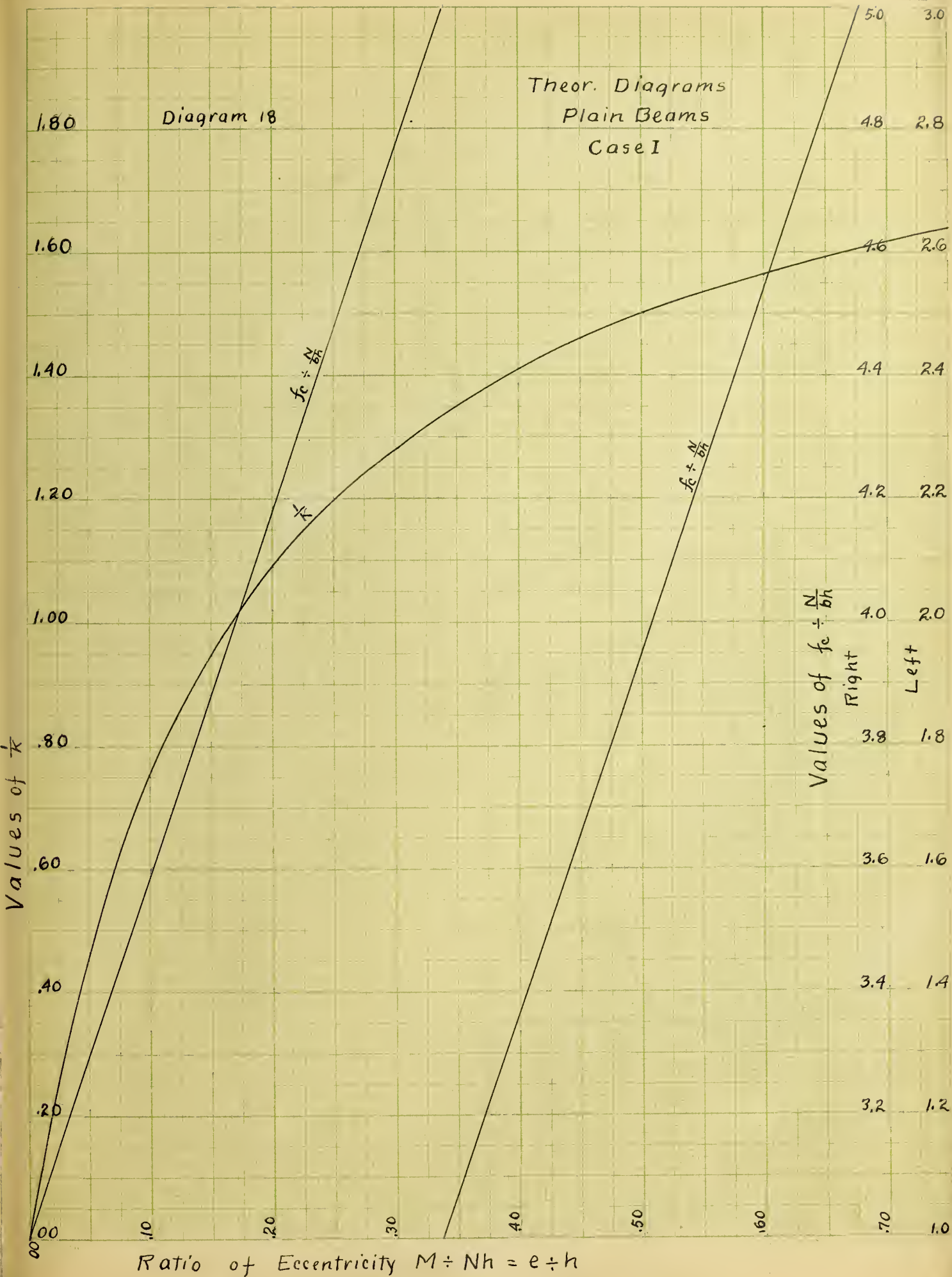
3

Diagram B

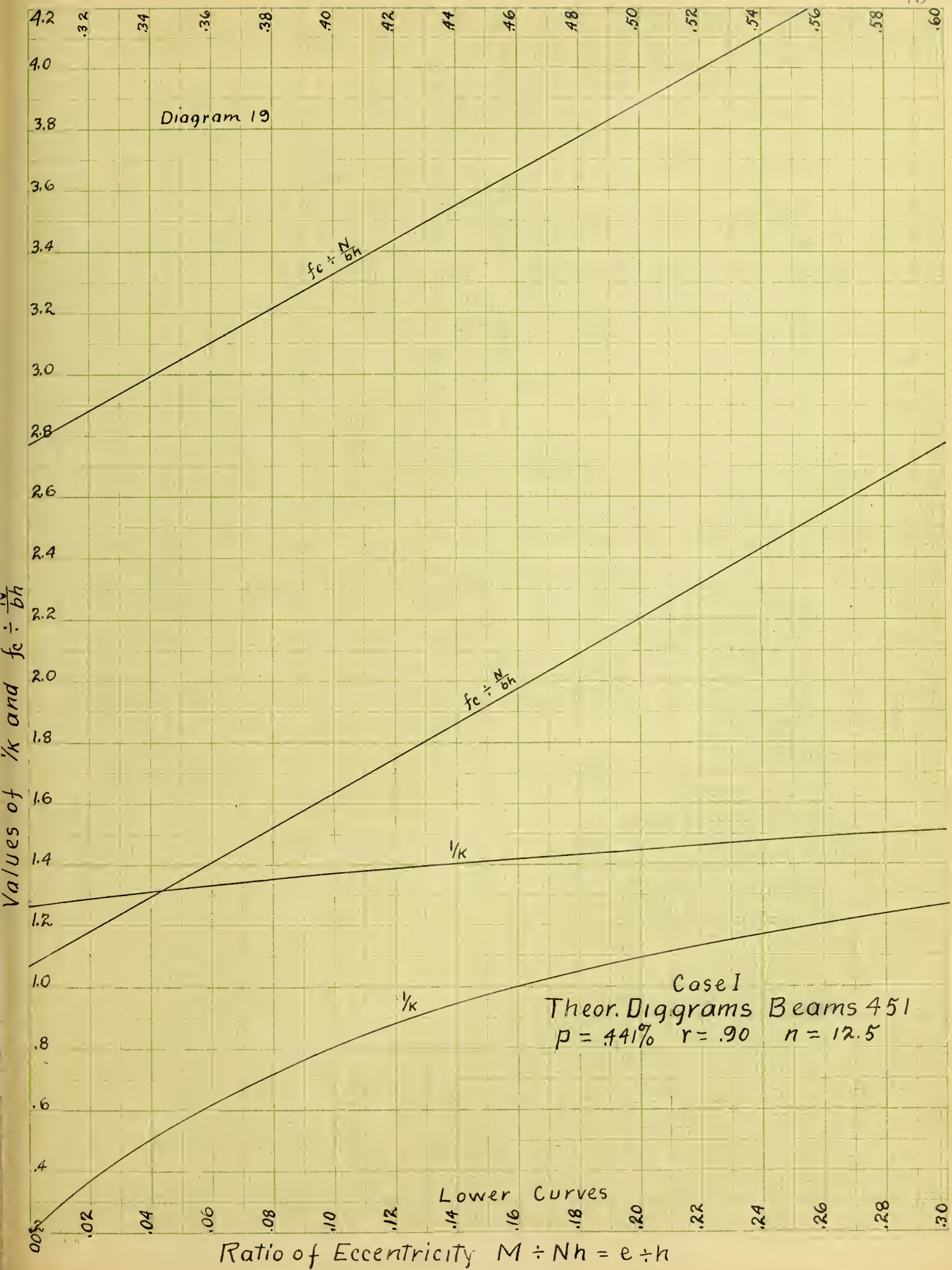
Unit Deformation in Upper Fiber
Center of Upper Roller $5\frac{1}{2}$ " above Beam.
Rollers 20.31" Apart Vertically.

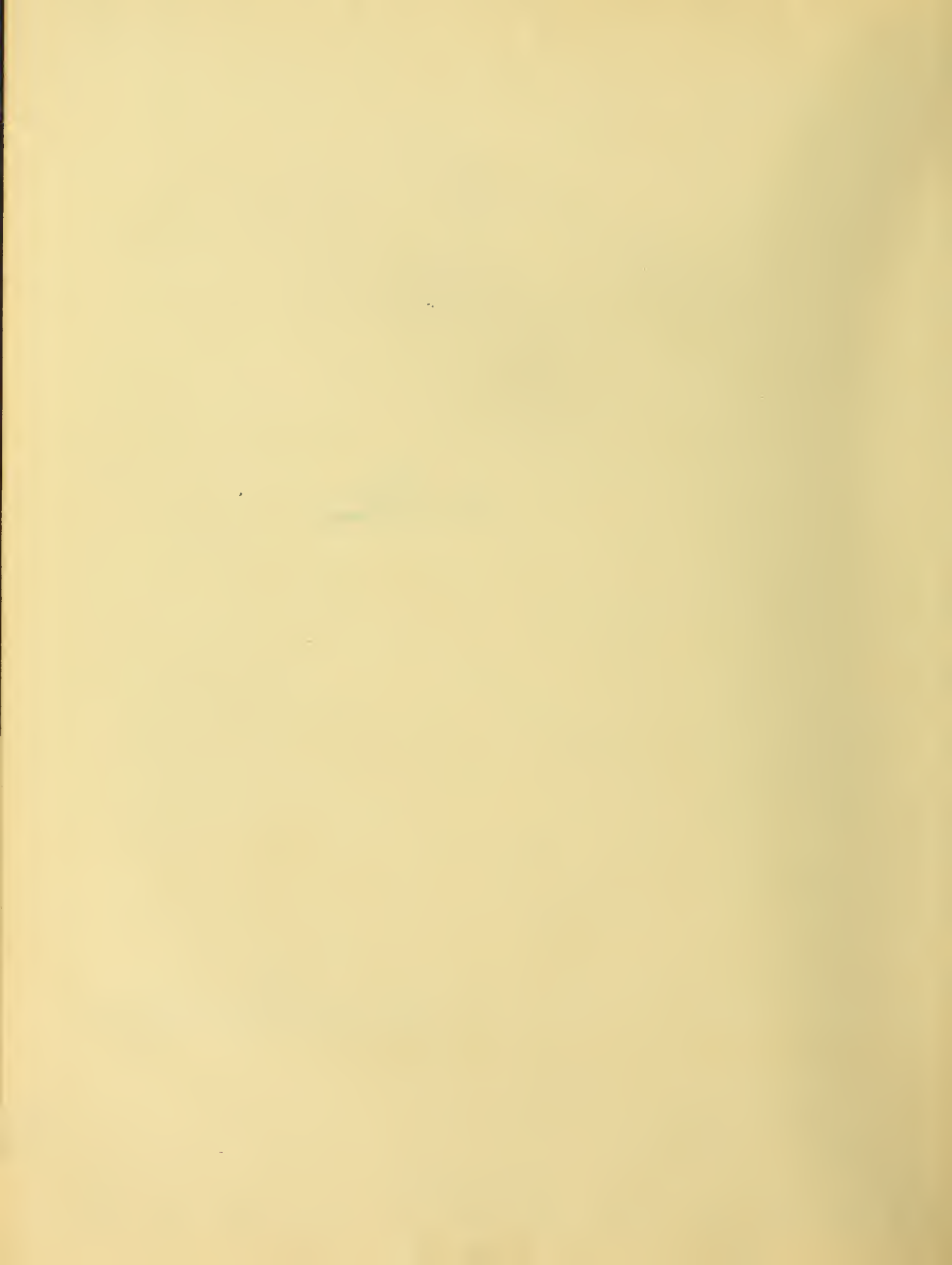
Gage Length = 40"

Lower Extensometer Reading ϵ_2









Values of $\frac{1}{k}$ and $f_c \div \frac{N}{bh}$

Upper Curves

Diagram 20

$f_c \div \frac{N}{bh}$

$f_c \div \frac{N}{bh}$

$\frac{1}{k}$

$\frac{1}{k}$

Case 1

Theor. Diagrams Beams 452

$p = .441\%$ $p' = .882\%$

$r = .90$ $r' = .13$ $n = 12.5$

Lower Curves

Ratio of Eccentricity $M \div Nh = e \div h$

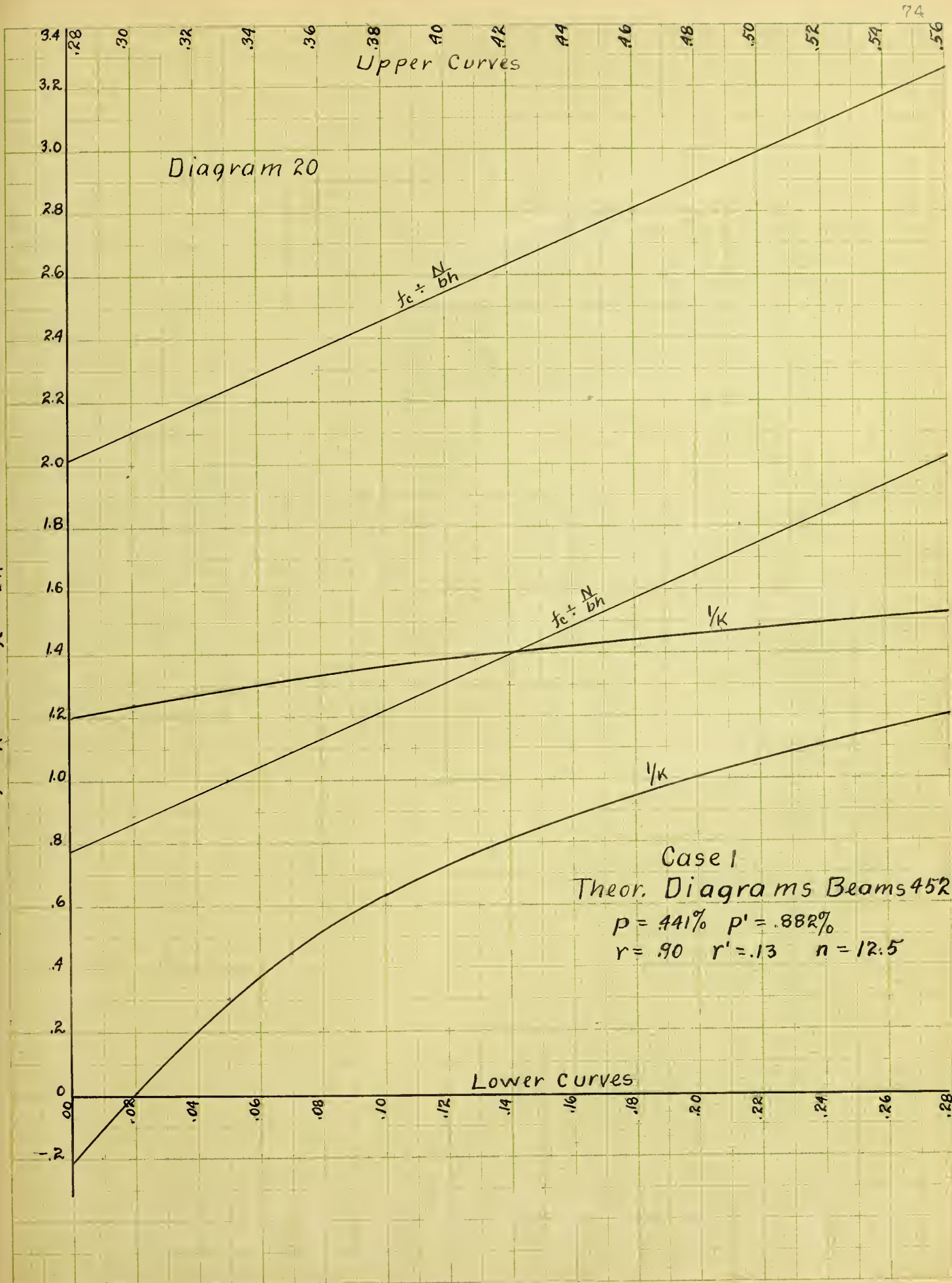


Diagram 21

Proportionate Depth of Neutral Axis, K

.20

.30

.40

.50

.60

.70

.80

.90

1.00

452.2

451

.60

1.00

1.40

1.80

2.20

2.60

3.00

457

452

Theor. Diagrams

Case II

Beams 452.2 $n = 12.5$ $p = .441\%$ $p' = .882$ $r = .90$ $r' = .13$ Beams 451. $n = 12.5$ $p = .441\%$ $r = .90$

Lower Curves

.15

.20

.25

.30

.35

.40

.45

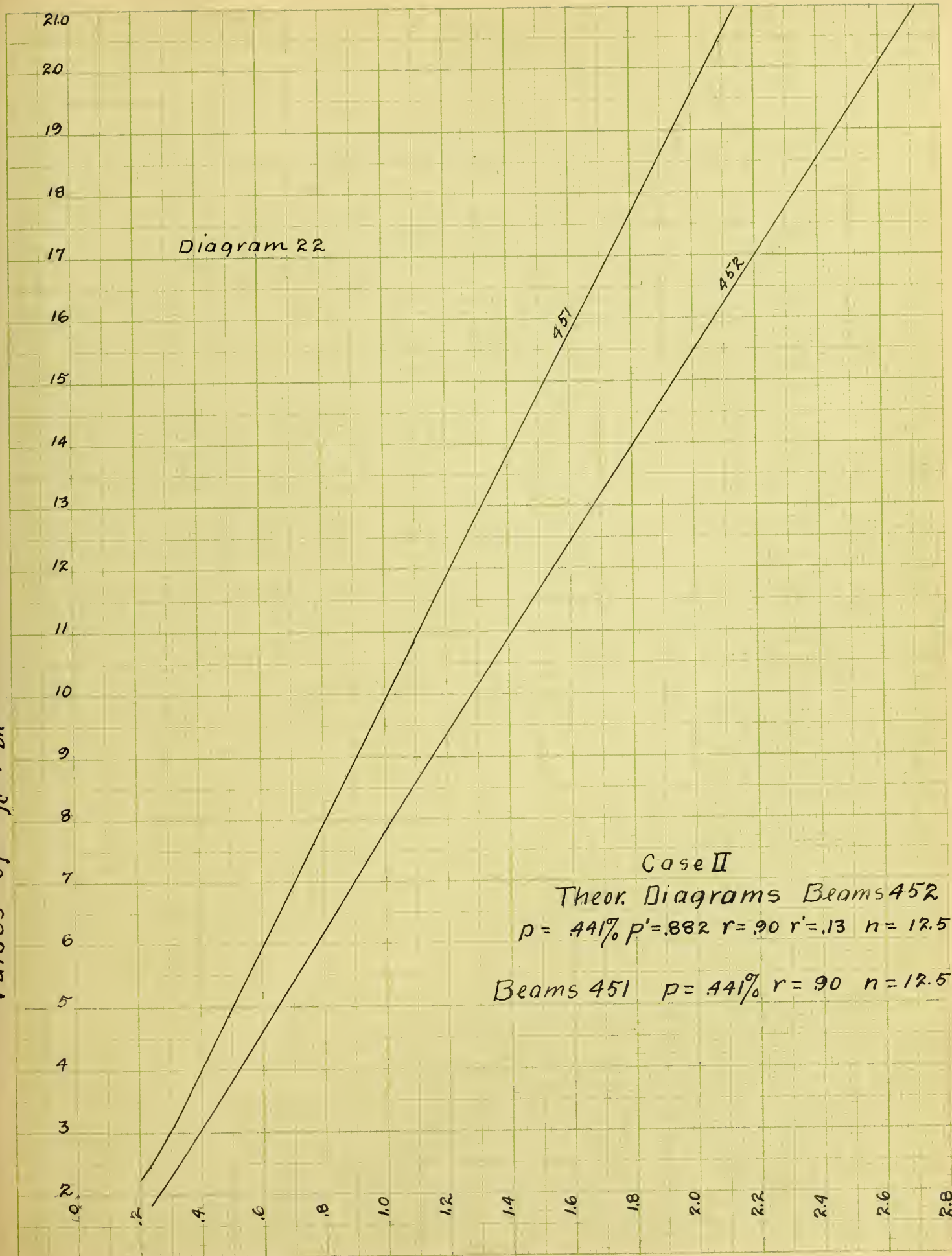
.50

.55

.60

Ratio of Eccentricity $M \div Nh = e \div h$

Values of $f_c \div \frac{N}{bh}$



Ratio of Eccentricity $M \div Nh = e \div h$

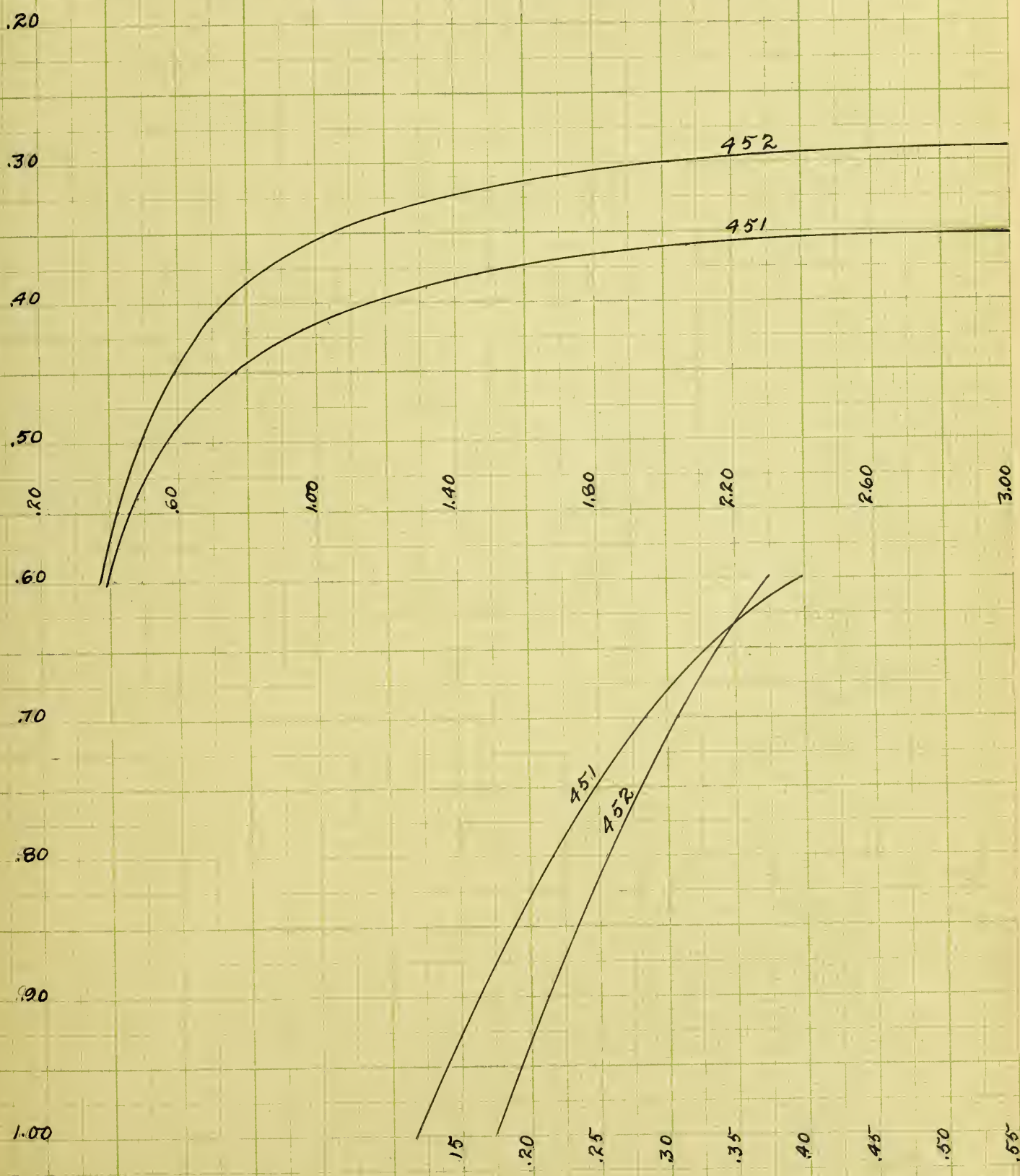
Case III
Theor. Diagrams

Beams 452. $p = 441\%$ $p' = .882\%$ $r = 90$ $r' = .13$ $n = 125$

Diagram 23

Beams 451 $p = 441\%$ $r = .90$ $n = 12.5$

Proportionate Depth of Neutral Axis. K



Ratio of Eccentricity $M \div Nh = e \div h$

Case III

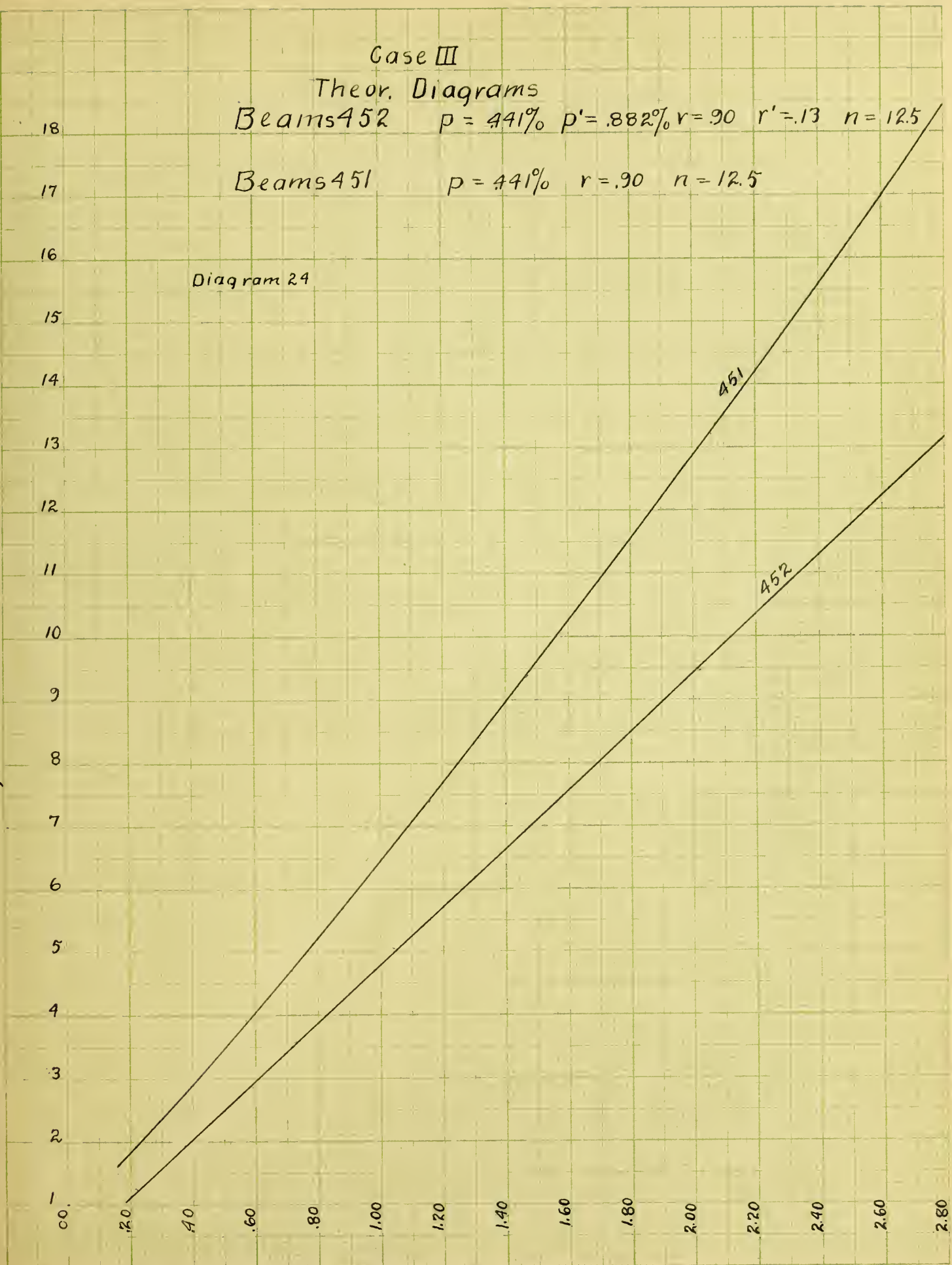
Theor. Diagrams

Beams 452 $p = 441\%$ $p' = .882\%$ $r = .90$ $r' = .13$ $n = 12.5$

Beams 451 $p = 441\%$ $r = .90$ $n = 12.5$

Diagram 24

Values of $f_c \div \frac{N}{bh}$



Ratio of Eccentricity $M \div Nh = e \div h$

Ratio of Eccentricity $M \div Nh$ (Upper Curves)

79

Proportionate Depth of Neutral Axis K

Diagram 25

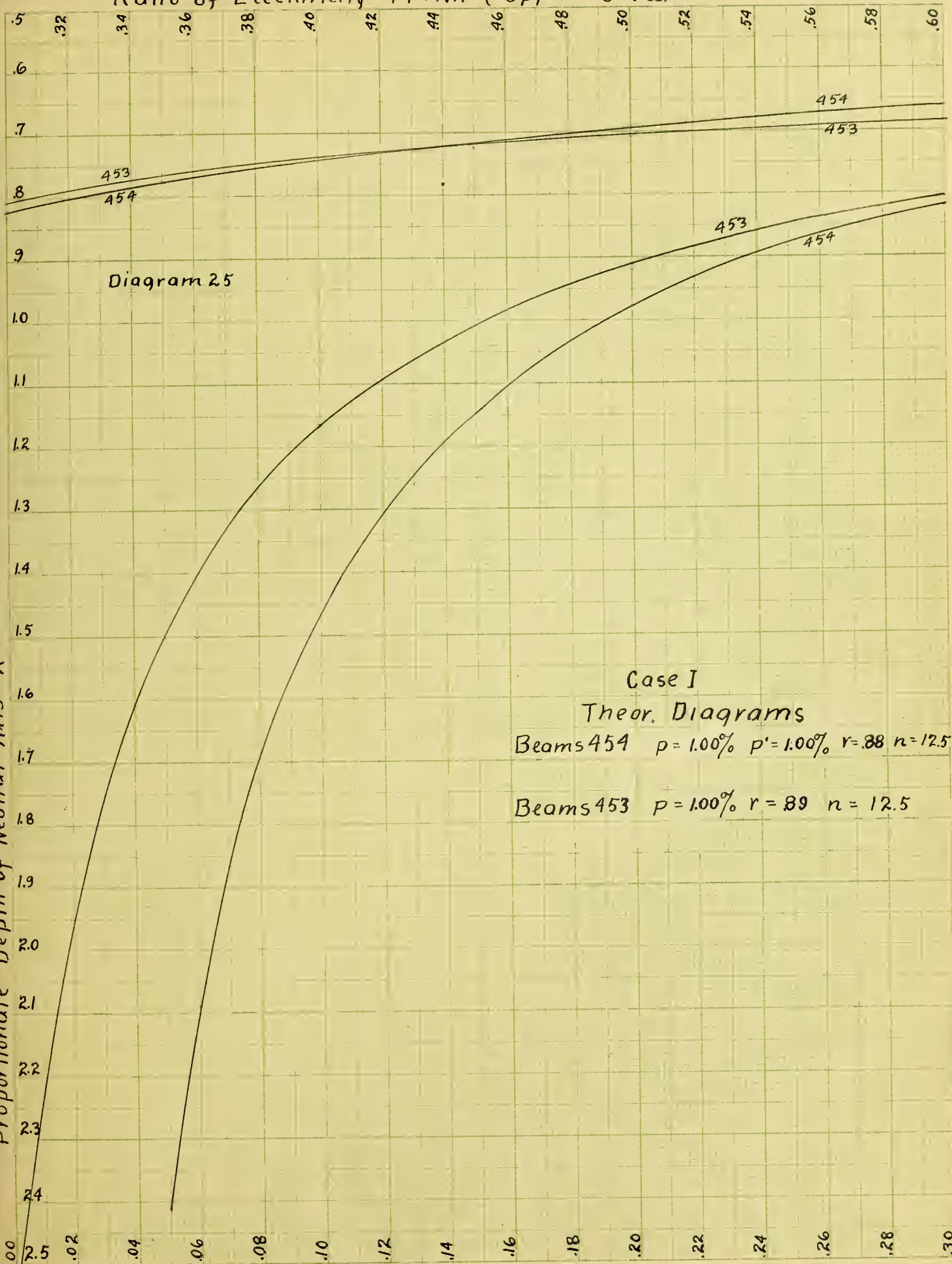
Case I

Theor. Diagrams

Beams 454 $p = 1.00\%$ $p' = 1.00\%$ $r = .88$ $n = 12.5$

Beams 453 $p = 1.00\%$ $r = .89$ $n = 12.5$

Ratio of Eccentricity $M \div Nh$ (Lower Curves)



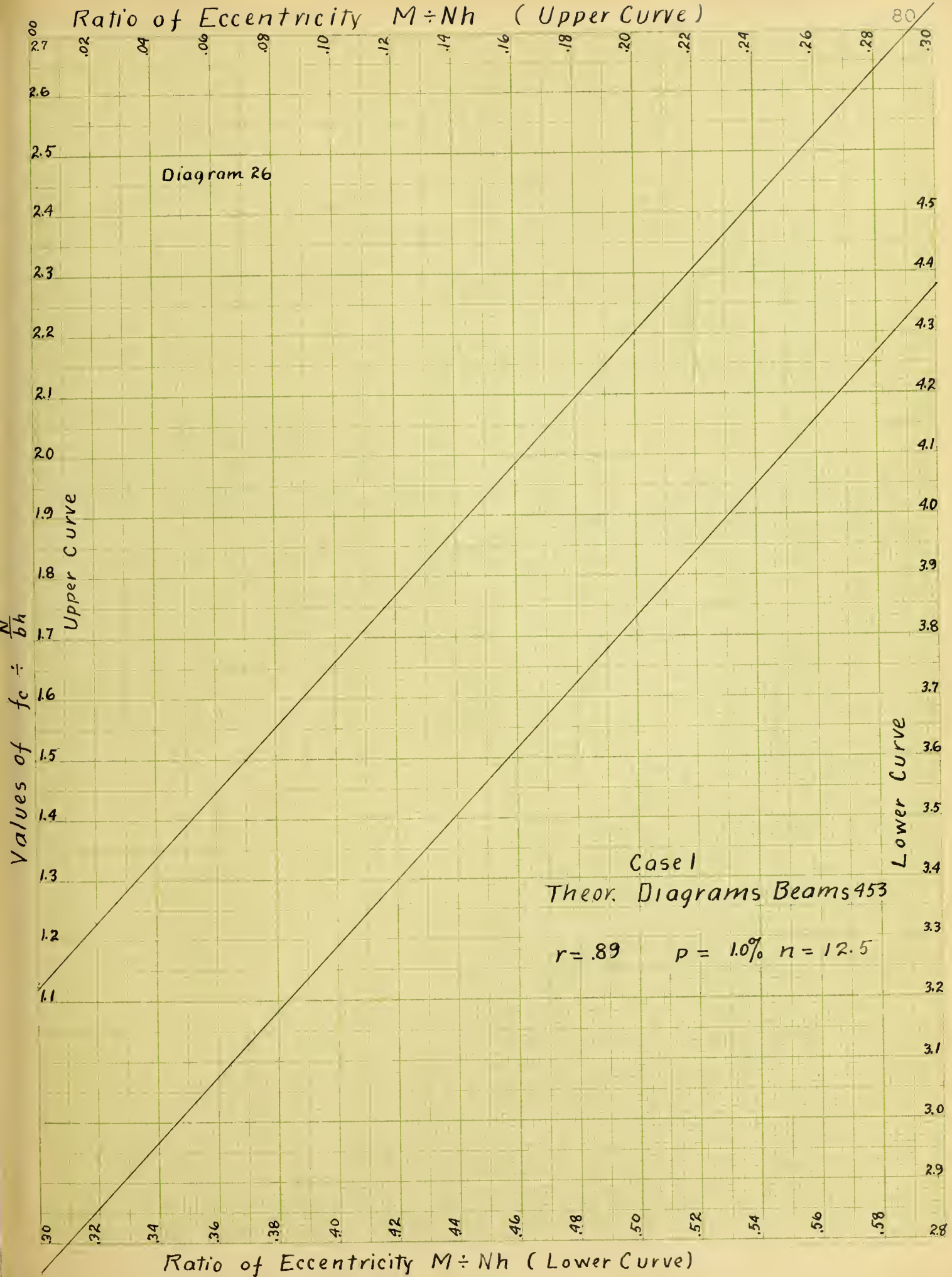
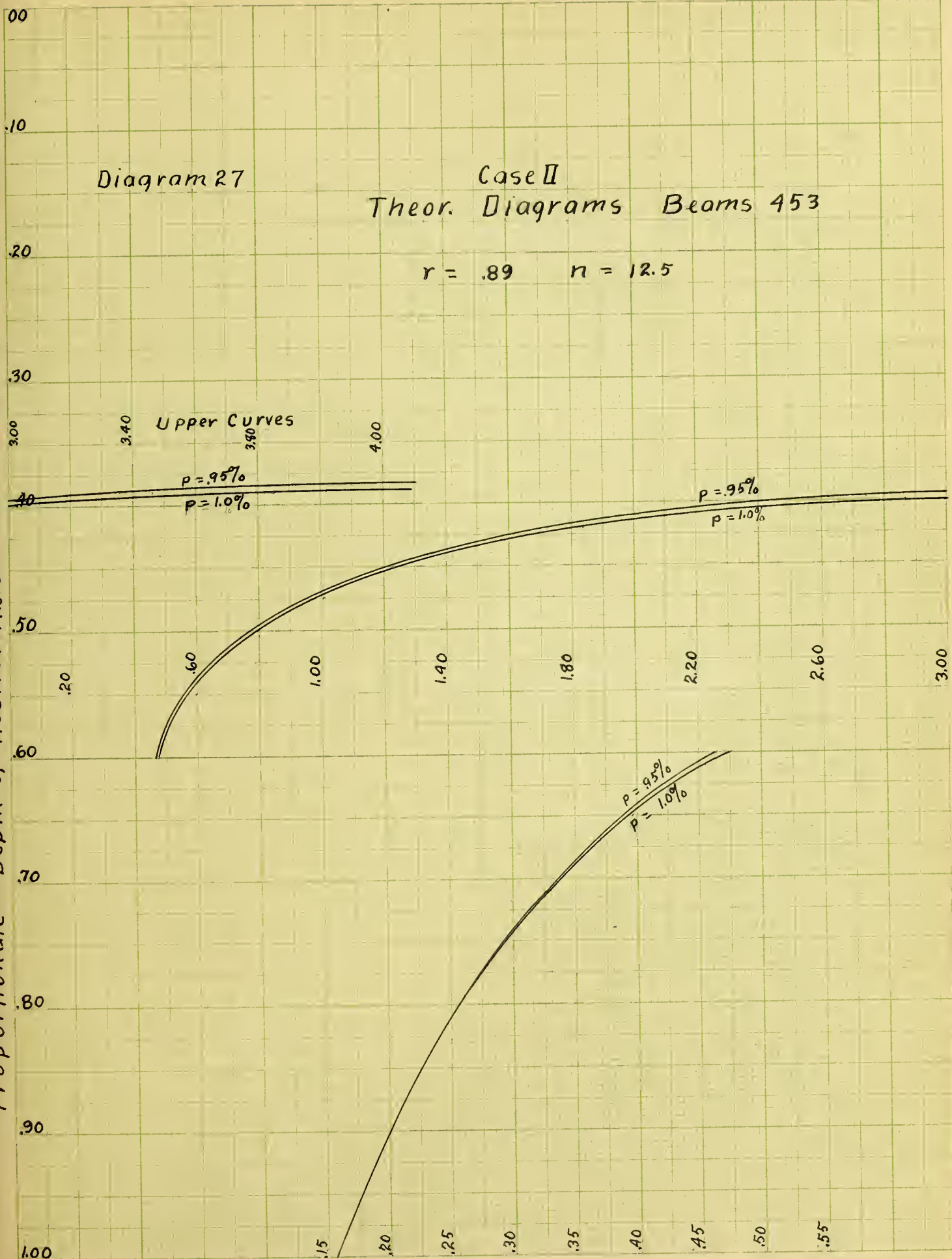


Diagram 27

Case II

Theor. Diagrams Beams 453

$$r = .89 \quad n = 12.5$$

Proportionate Depth of Neutral Axis K 

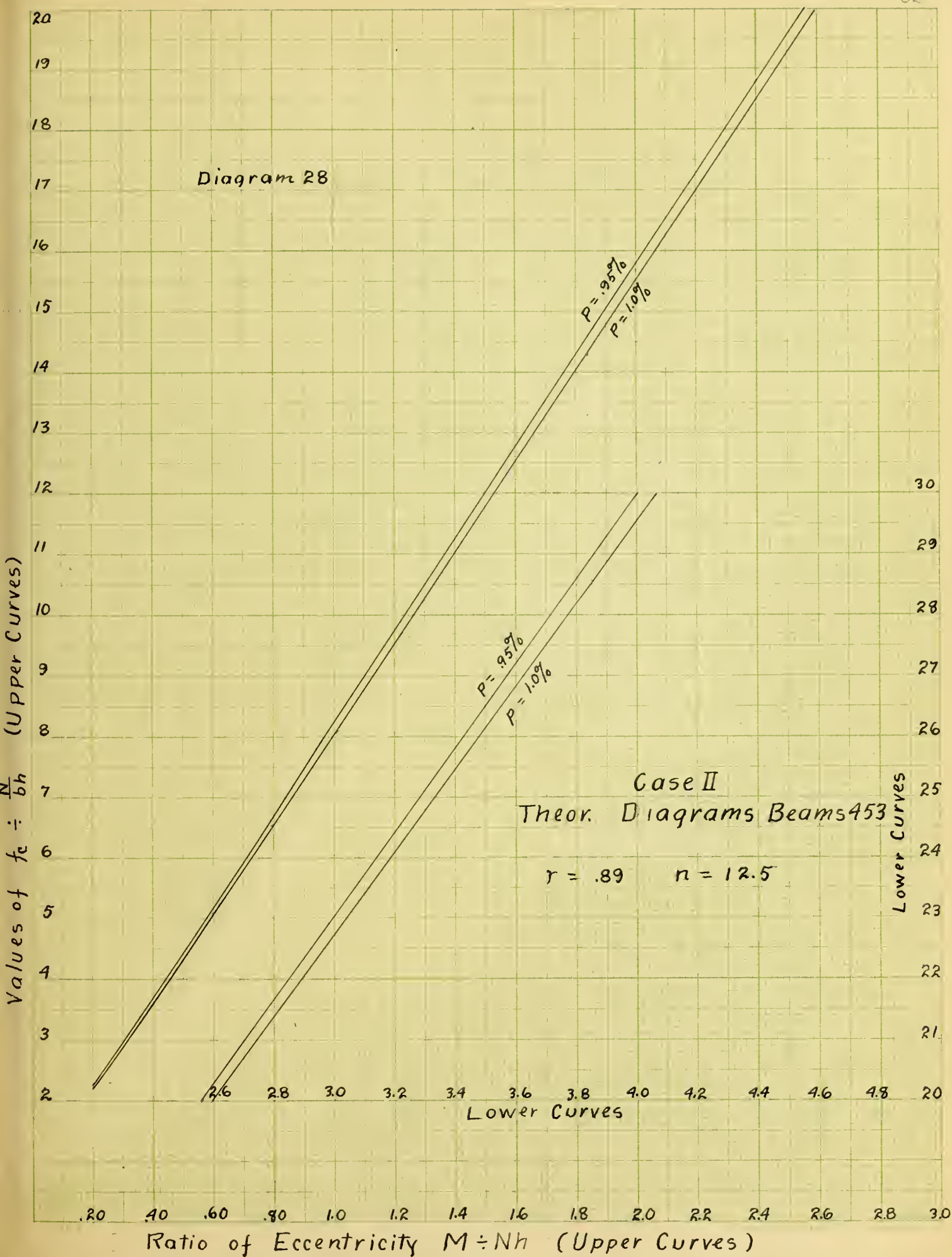


Diagram 29

Case III
Theor. Diagrams Beams 453

$$r = .89 \quad n = 12.5$$

Proportionate Depth of Neutral Axis, K

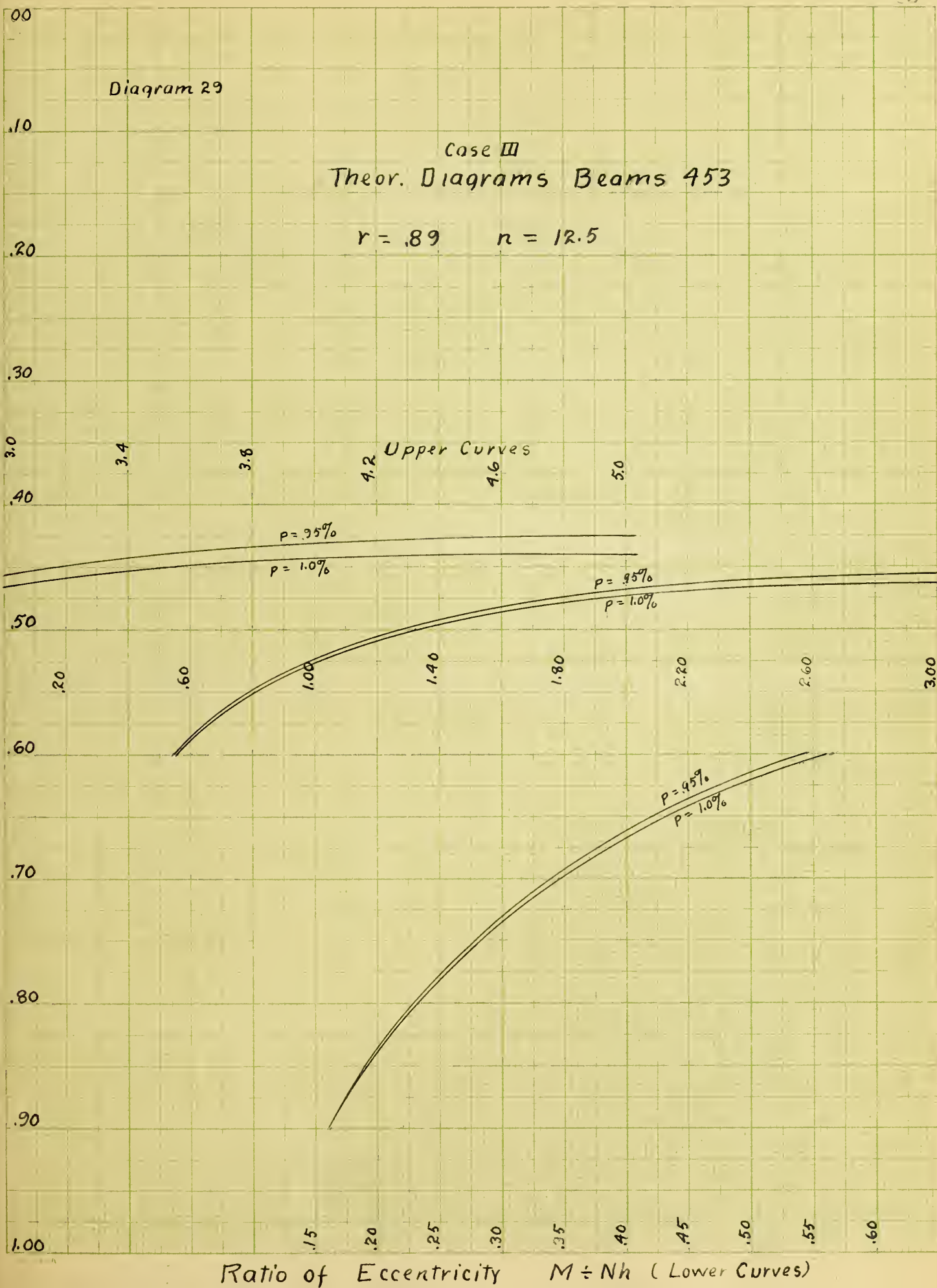
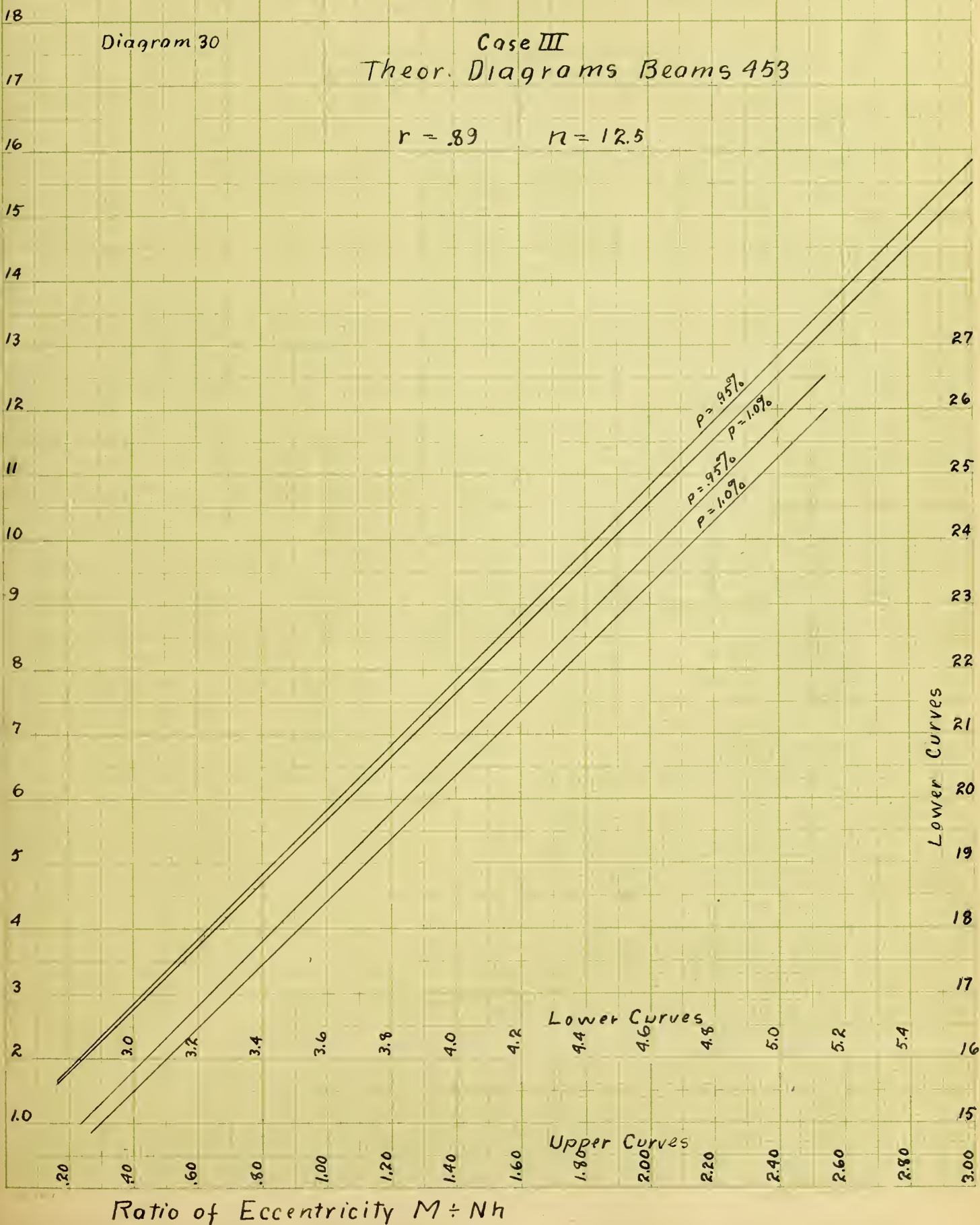


Diagram 30

Case III
Theor. Diagrams Beams 453 $r = .89$ $n = 12.5$ Values of $f_c \div b h$ (Upper Curves)

Lower Curves



Case I
Theor. Diagrams for Beams 454

Diagram 31

$r = .88 = (1 - r')$ $n = 12.5$

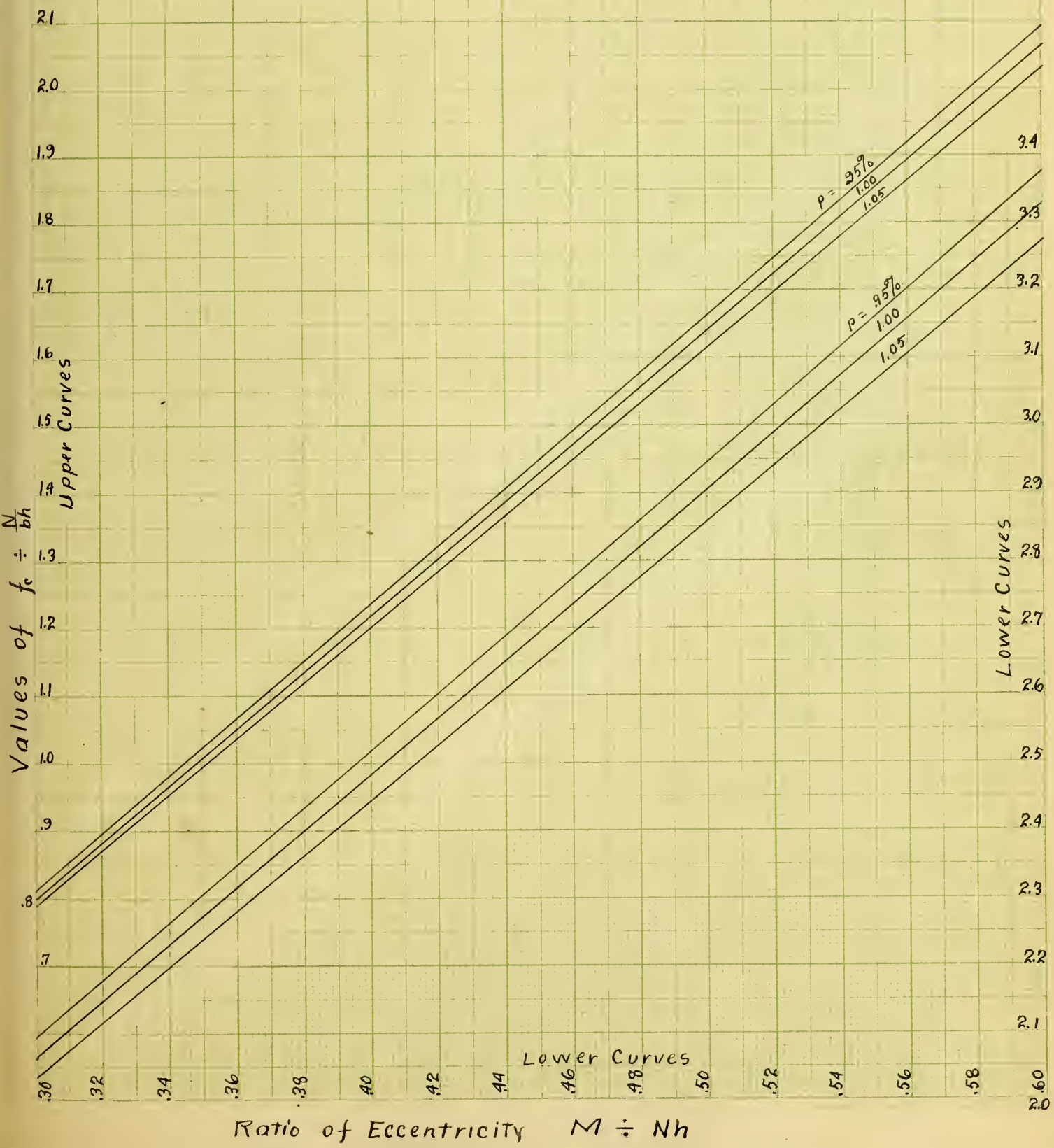


Diagram 32

Case II
Theor. Diagrams for Beams 454

$$r = .88 = (1 - r') \quad n = 12.5$$

Proportionate Depth of Neutral Axis. K

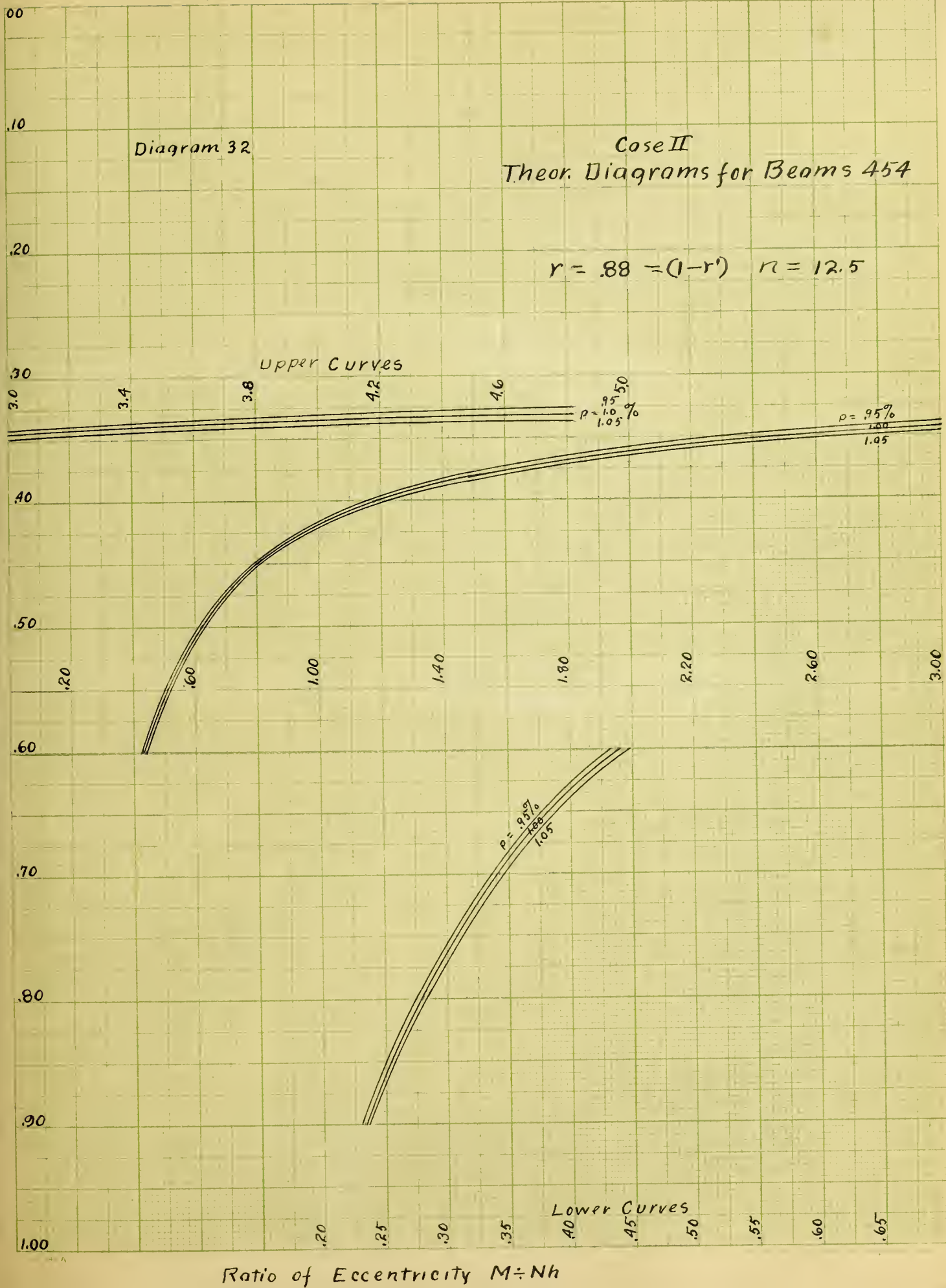
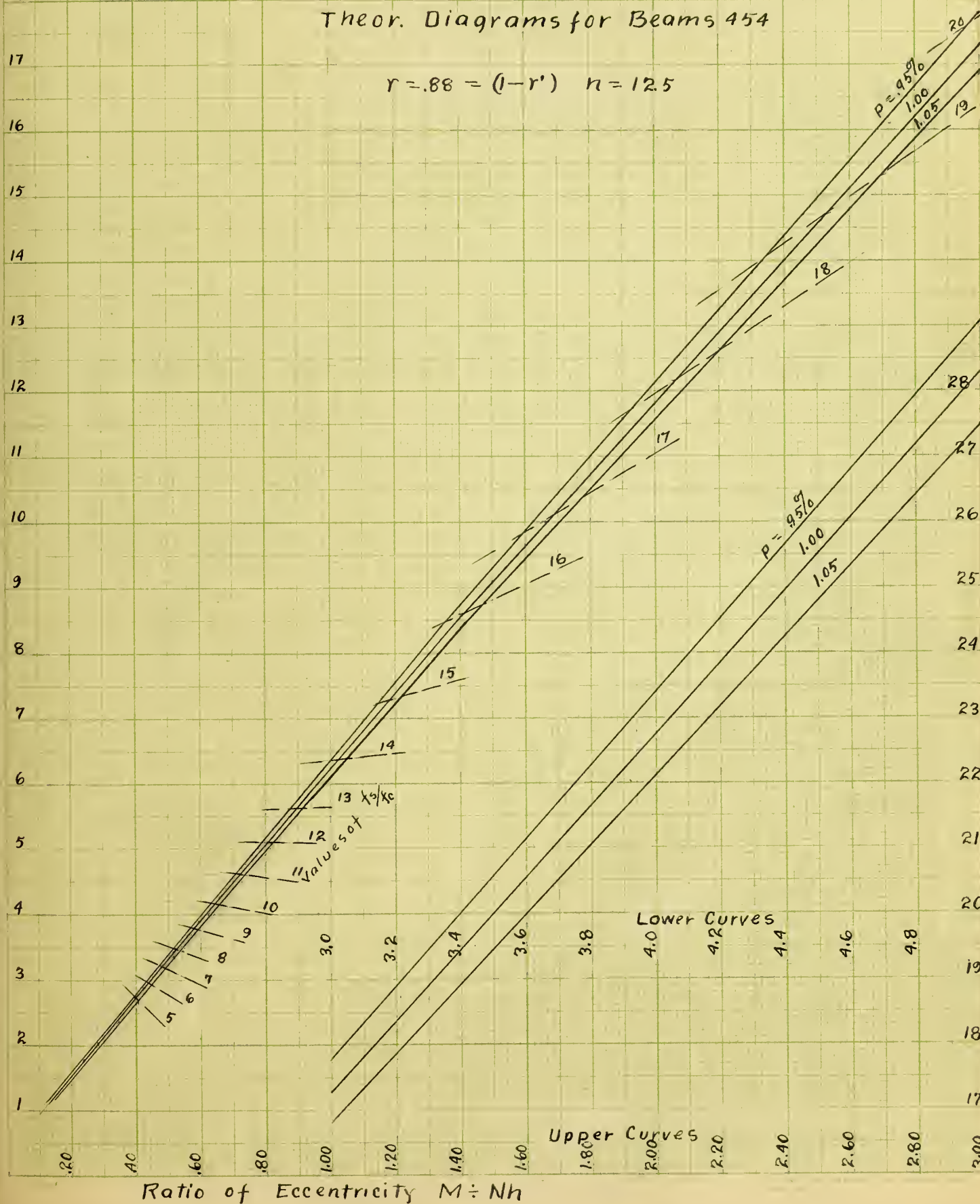


Diagram 33

Case II

Theor. Diagrams for Beams 454

$$r = .88 = (1 - r') \quad n = 12.5$$



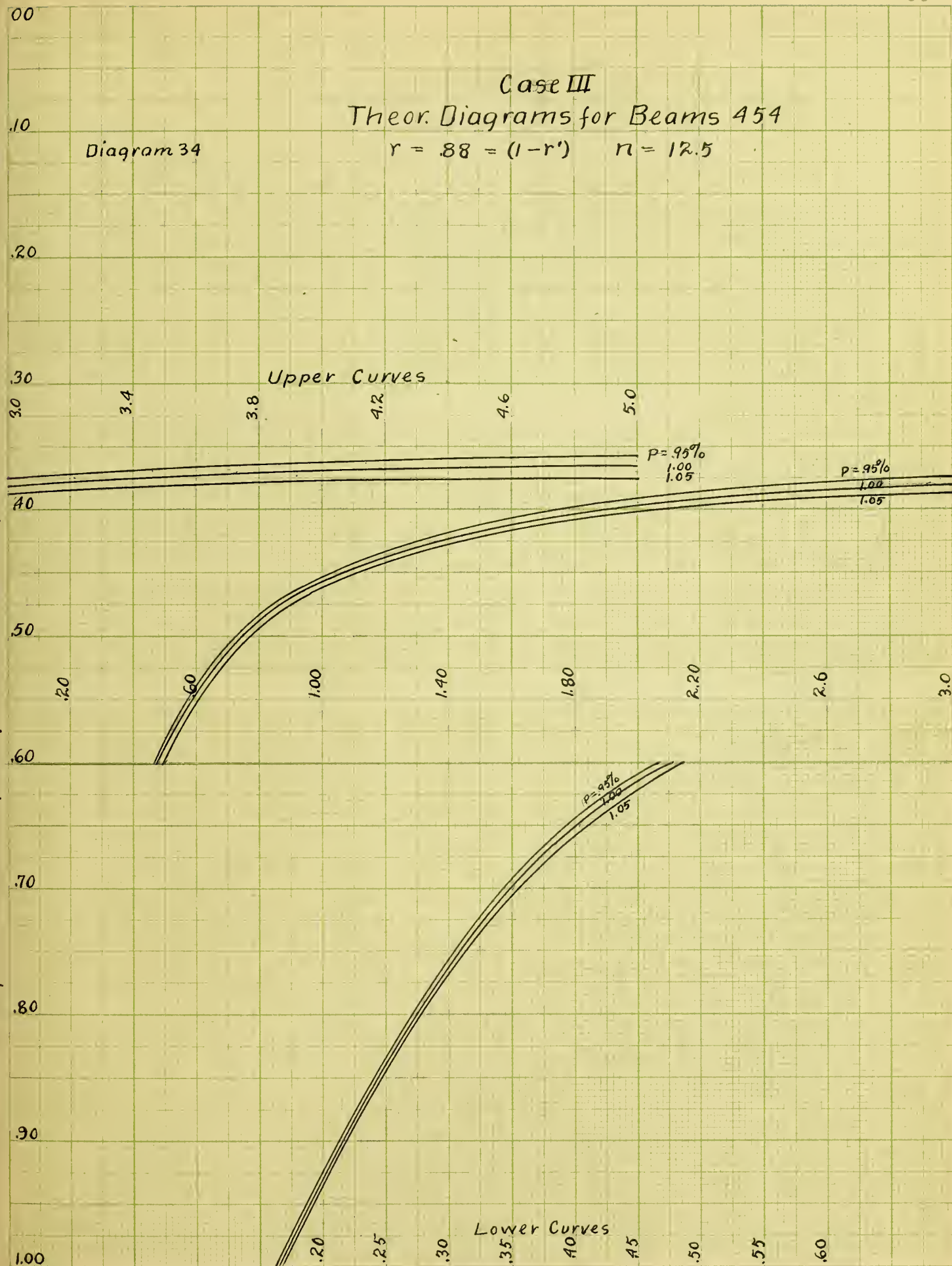
Case III Theor. Diagrams for Beams 454

Diagram 34

$$r = .88 = (1 - r') \quad n = 12.5$$

Proportionate Depth of Neutral Axis, K

Upper Curves



Ratio of Eccentricity $M \div Nh$

Lower Curves

Case III
Theor. Diagrams for Beams 454
 $r = .88 = (1-r')$ $n = 12.5$

Diagram 35

18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0

$P = .95\%$
1.00
1.05

Upper Curves

3.4

3.8

4.2

4.6

5.0

$P = .95\%$
1.00
1.05

31

33
32

30

25

20

15

10

Ratio of Eccentricity, $M \div Nh$

Lower Curves

values of $I_c \div bh^3$

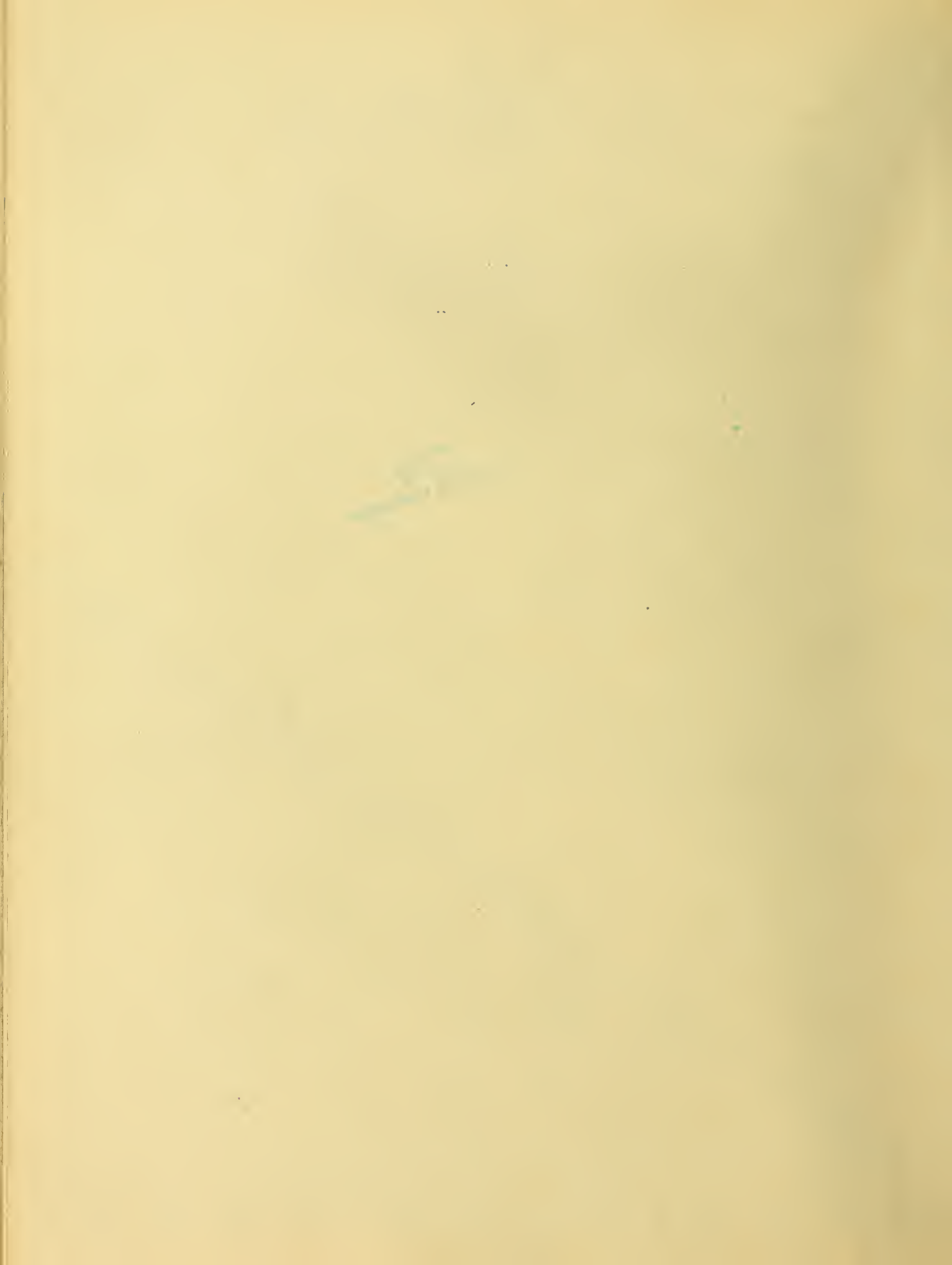


Diagram 37

Beam 450.3
Plain Concrete
Exp. Curve
Theor. "

Depth of Neutral Axis

Thrust in lb. per sq. in. of Section

103 102

Stress in Concrete lb. per sq. in.

Upper Fiber

Lower Fiber

Crack Formed

Equivalent 1/3 Pt. Load in Pounds

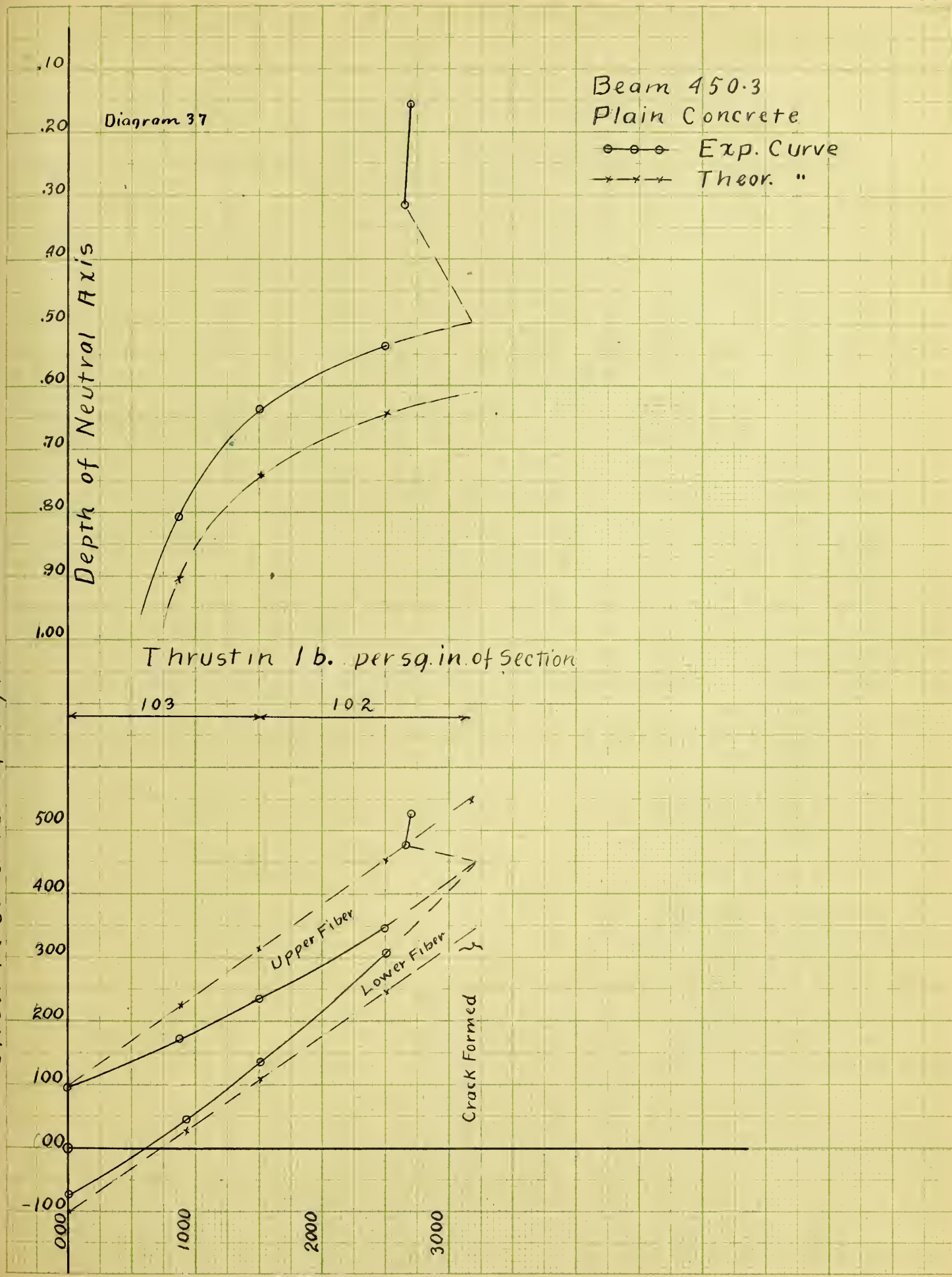


Diagram 36

Beam 450.1
Plain Concrete
Exp. Curve
Theor. "

Depth of Neutral Axis

.50
.60
.70
.80
.90
1.00

Thrust in lb. per sq. in. of Section

101

101

Repetition

Stress in Concrete Lb. per sq. in.

500
400
300
200
100
00
100

Upper Fiber

Lower Fiber

Upper Fiber

Lower Fiber

Failure on
Taking off End Load

Equivalent $\frac{1}{3}$ Pt. Load in Pounds

000

1000

2000

000

1000

2000

3000

Diagram 38

Beam 450-4 Plain Concrete

Depth of
Neutral Axis

Avg. Thrust lb. per sq. in. of Section

Exp. Curve
Theor. "

Repetition

Upper Fiber

Lower Fiber

{ Beam Fractured upto Neutral Axis

Equivalent $\frac{1}{3}$ Pt. Load in Pounds

Stress in Concrete lb. per sq. in.

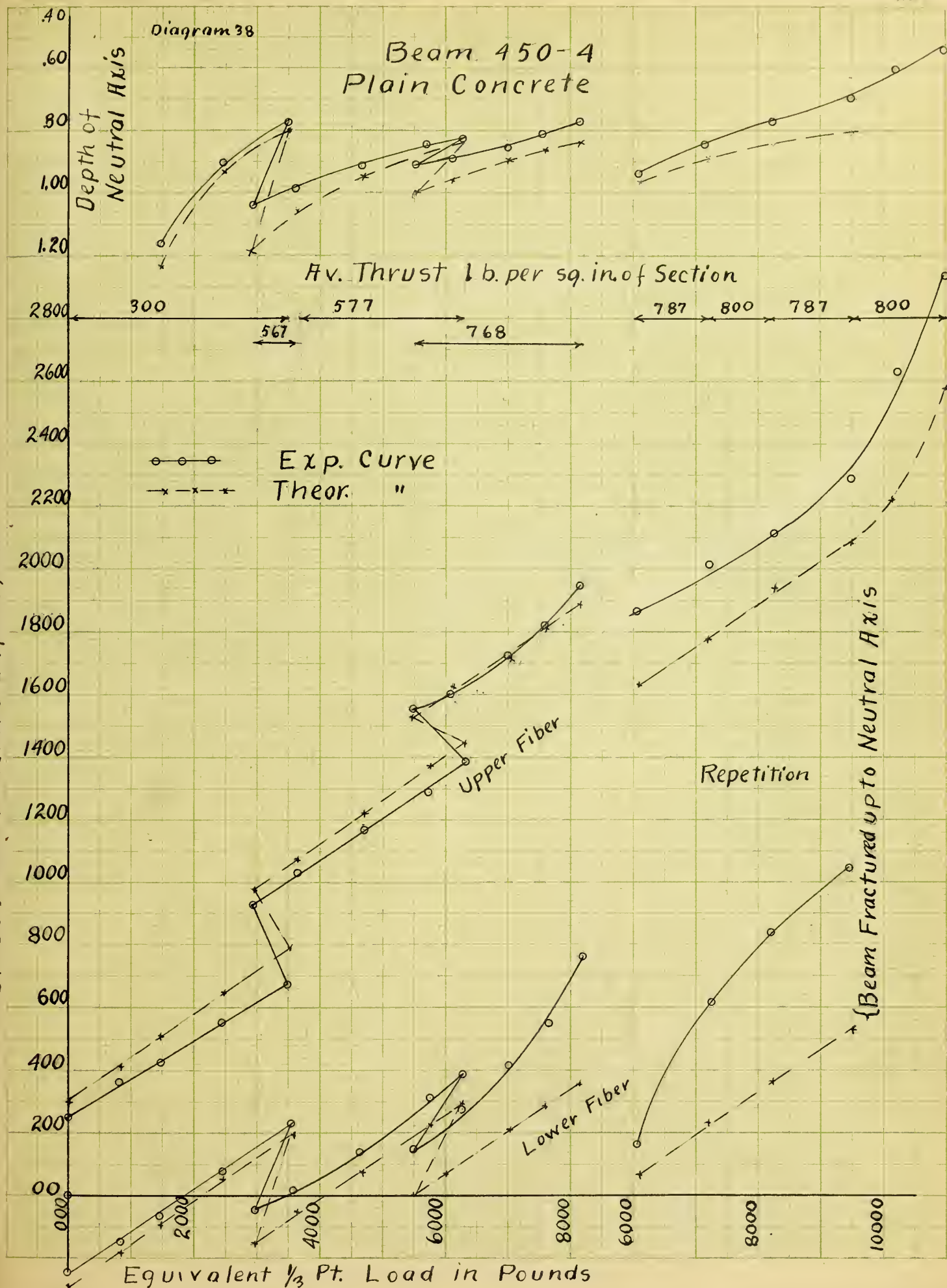


Diagram 39

Beam 450.5
Plain Concrete
—○—○—○— Exp. Curve
—*—*—*— Theor. "

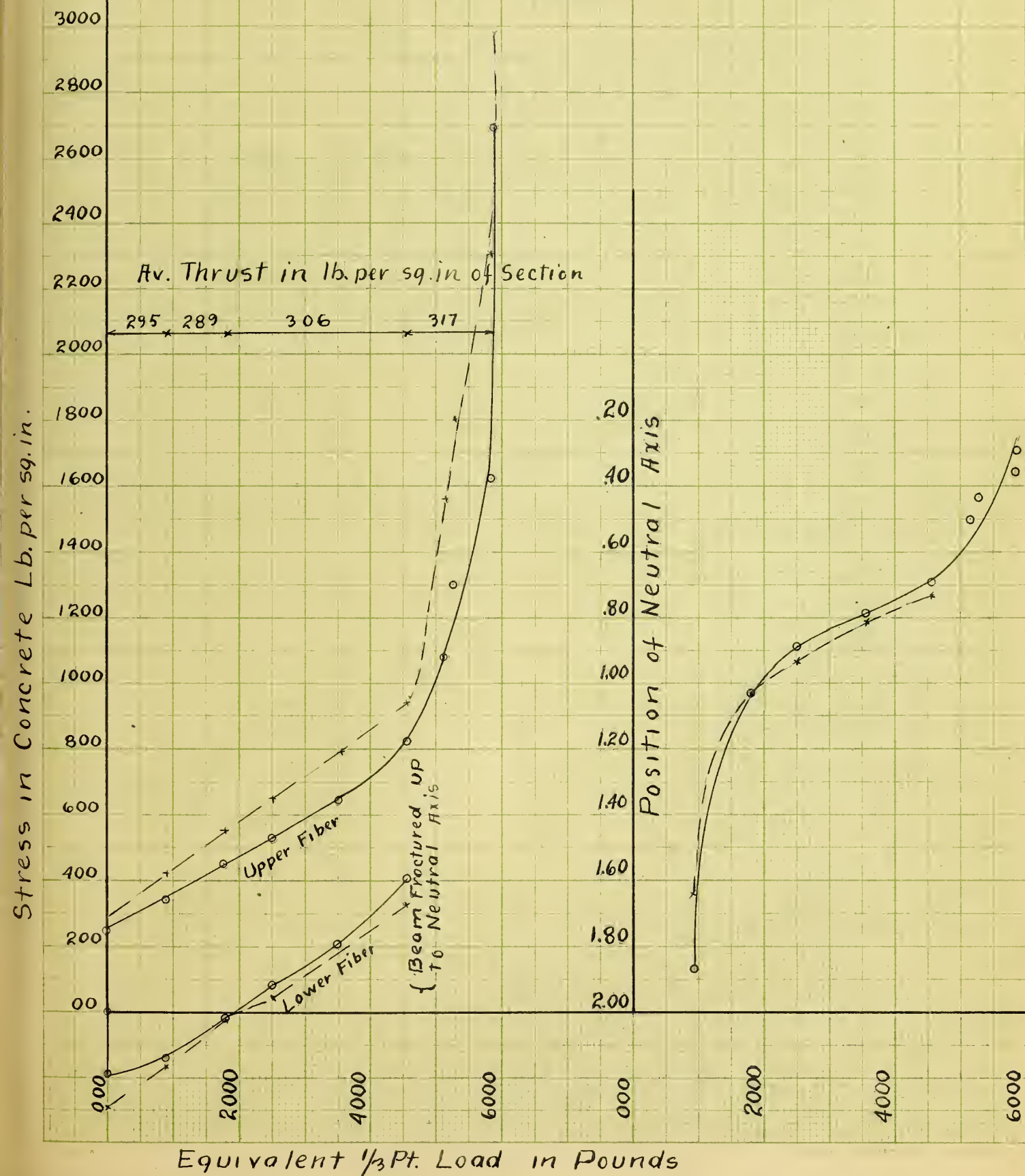


Diagram 40

Center Deflection
in Inches

Repetition

Simple Flexure

Beam 451.1
Reinforcement 2- $\frac{1}{2}$ " Φ Bottom
 $p = 40\%$ $r = .88$ $n = 12.5$
Exp. Curve
Failed by Tension in Steel
at 9418 Lbs Equiv. $\frac{1}{3}$ Pt. Load
Thrust = 10000 Lbs

Thrust = 100 lb per sq. in. of Section

Stress in Steel Lb. per sq. in.

10000

8000

6000

4000

2000

000

14000

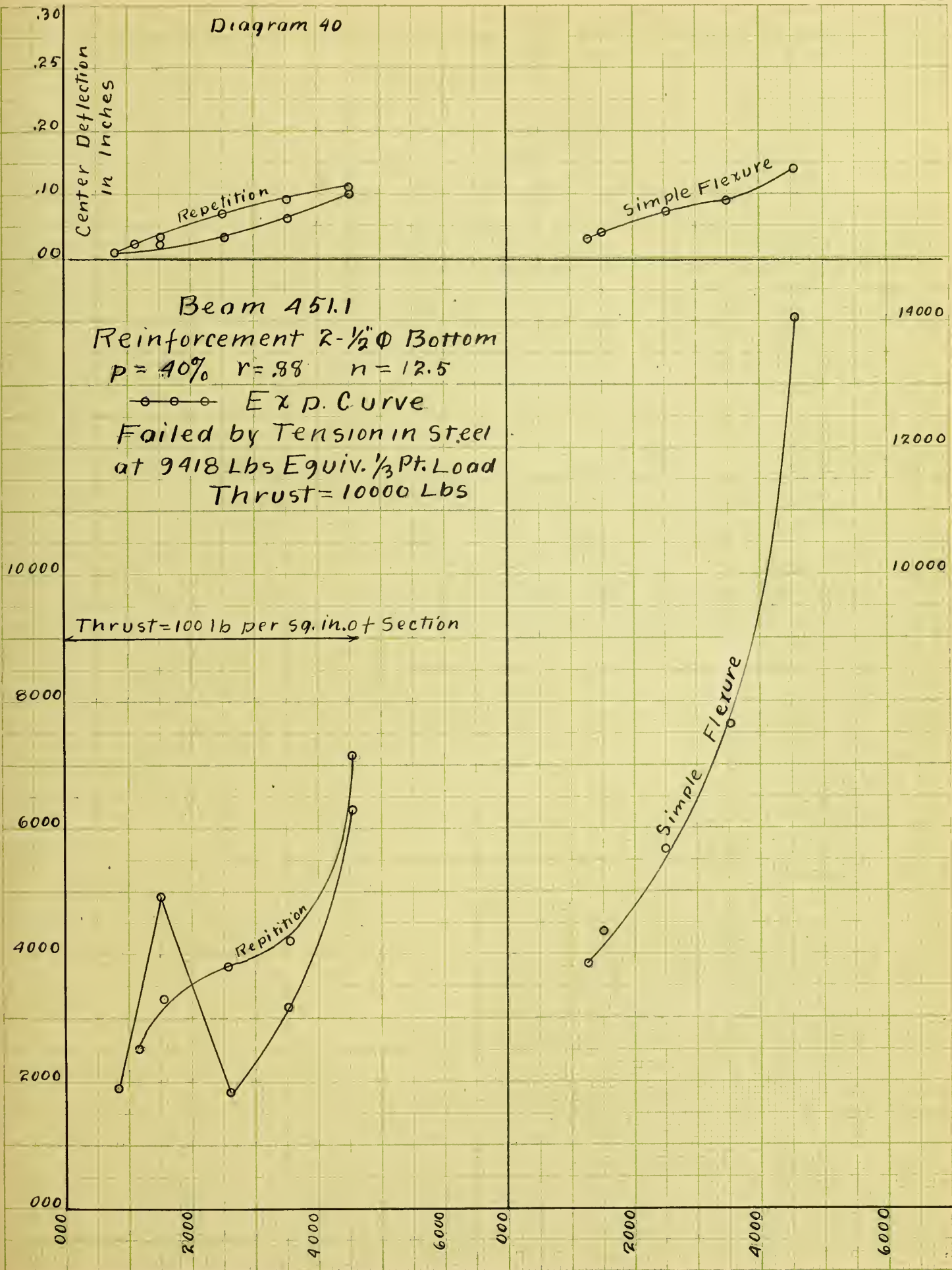
12000

10000

Simple Flexure

Equivalent $\frac{1}{3}$ Pt. Load in Pounds

Repetition



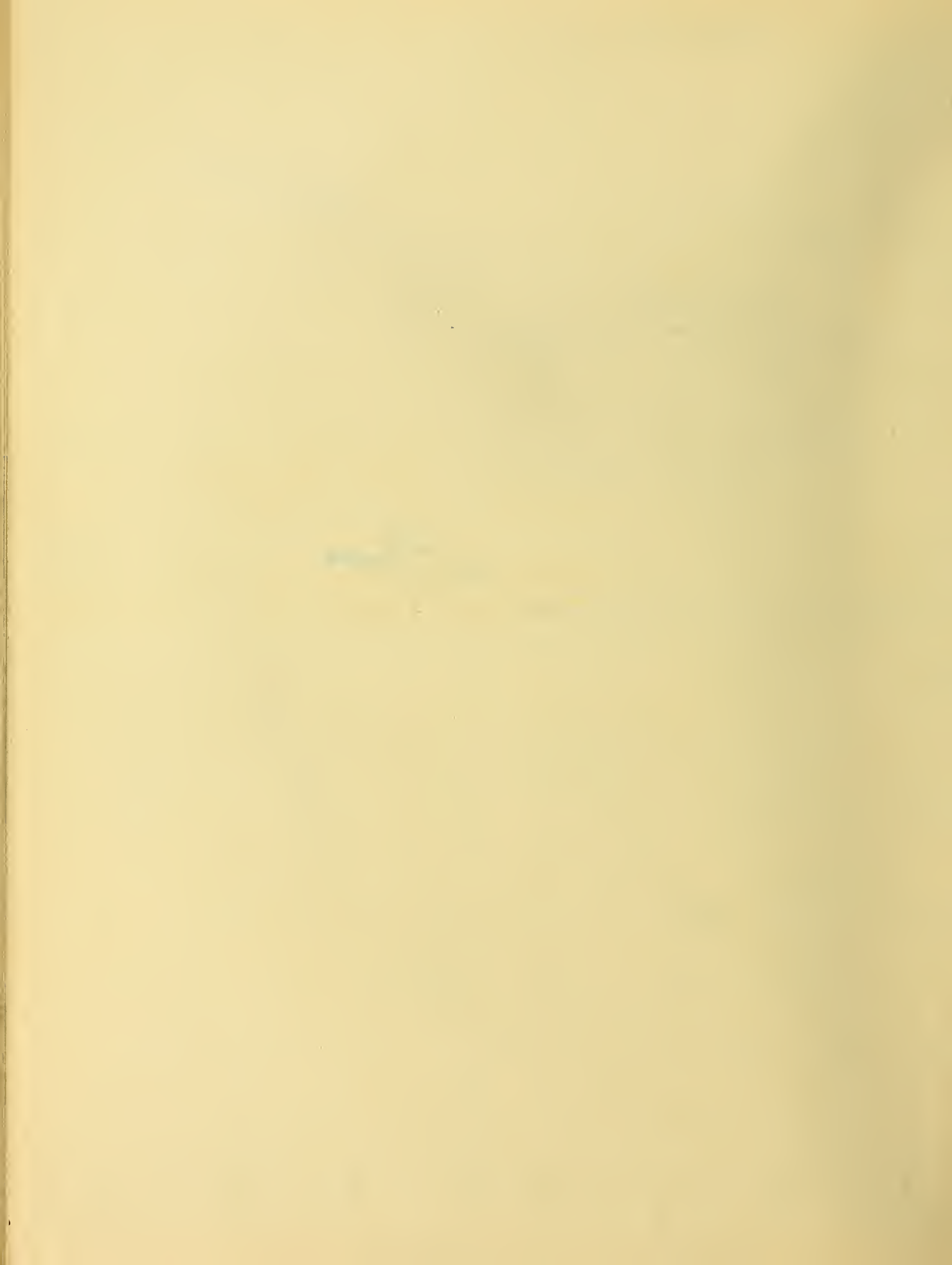


Diagram 41

Depth of Neutral Axis

.10
.20
.30
.40
.50
.60

Repetition

Beam 451.1

Reinforcement 2- $\frac{1}{2}$ " ϕ Bottom $p = .40\%$ $r = .882$ $n = 12.5$

Exp. Curve

Failed by Tension in the Steel at

9418 Lbs. Equiv. $\frac{1}{3}$ Pt. Load. Thrust = 10000 lb

800

Thrust = 100 lb. persq. in. of Section

600

400

200

00

Repetition

Simple Flexure

Equivalent $\frac{1}{3}$ Pt. Load in Pounds

6000

4000

2000

000

6000

4000

2000

000

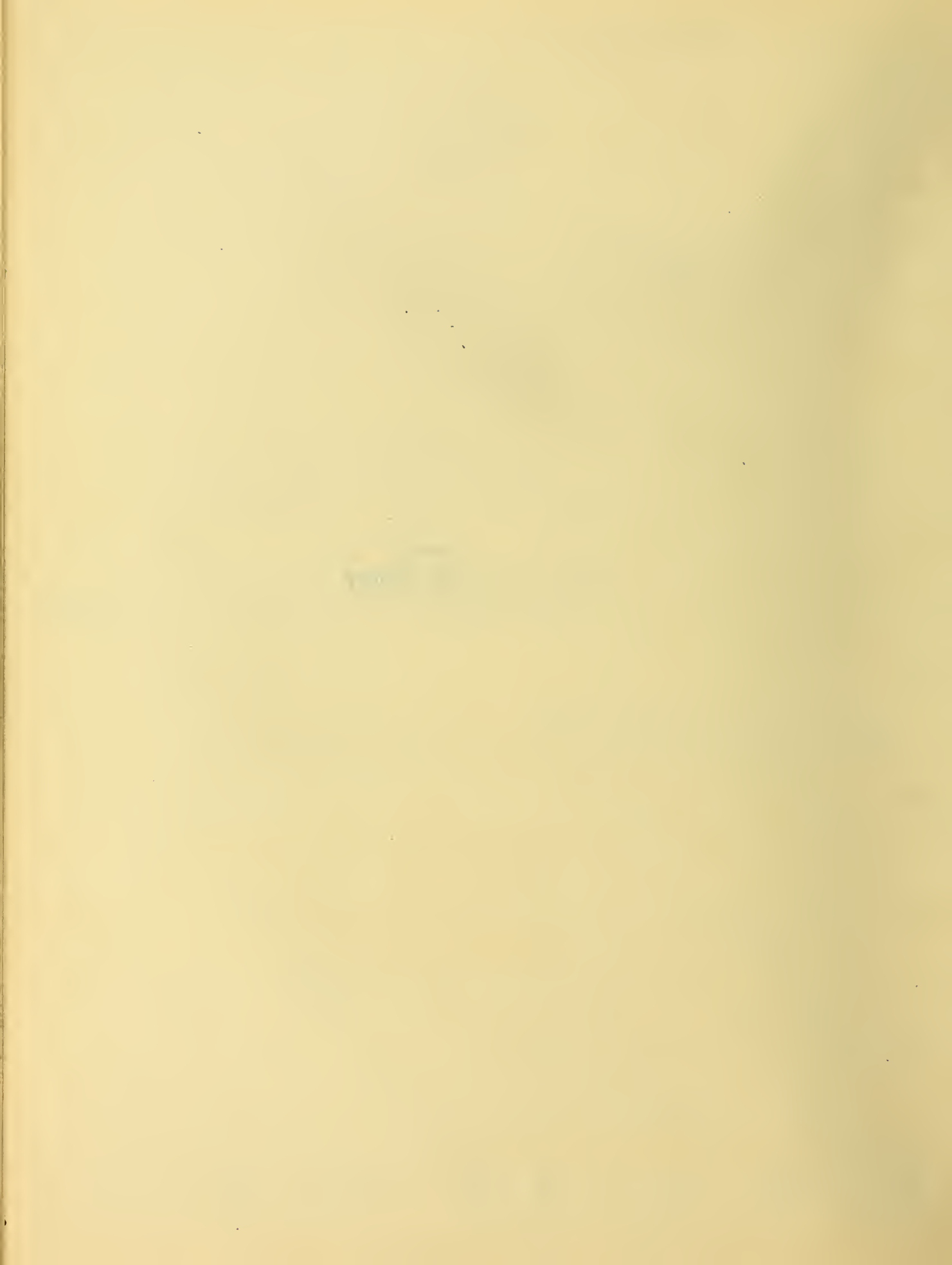
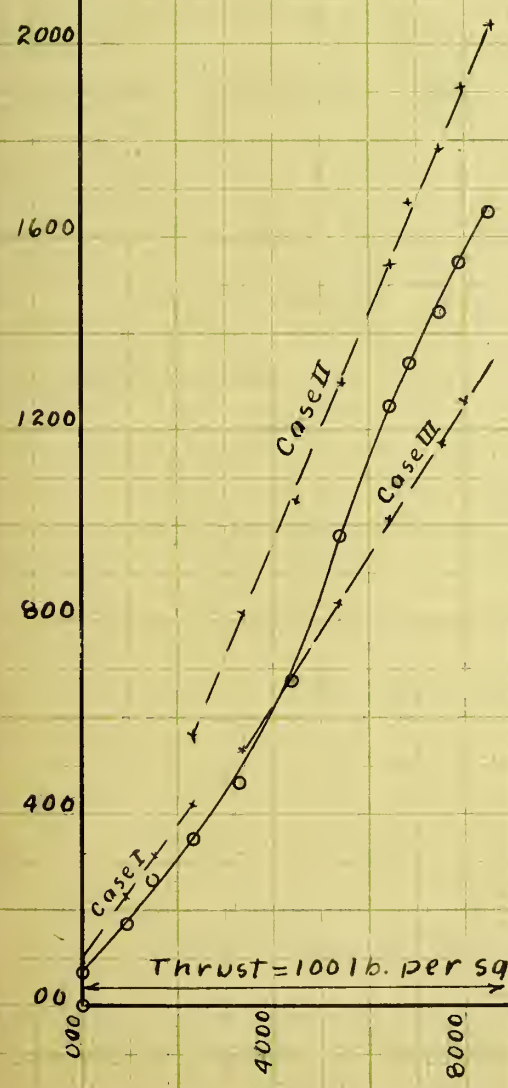
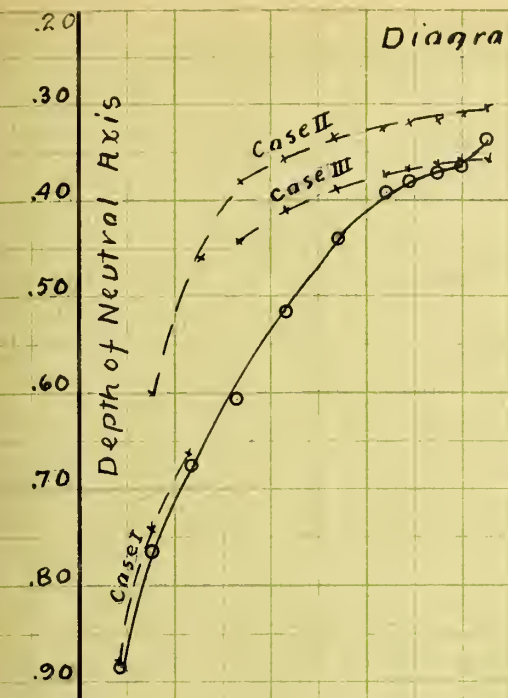


Diagram 42

Stress in Concrete Lb. per sq.in.

Depth of Neutral Axis



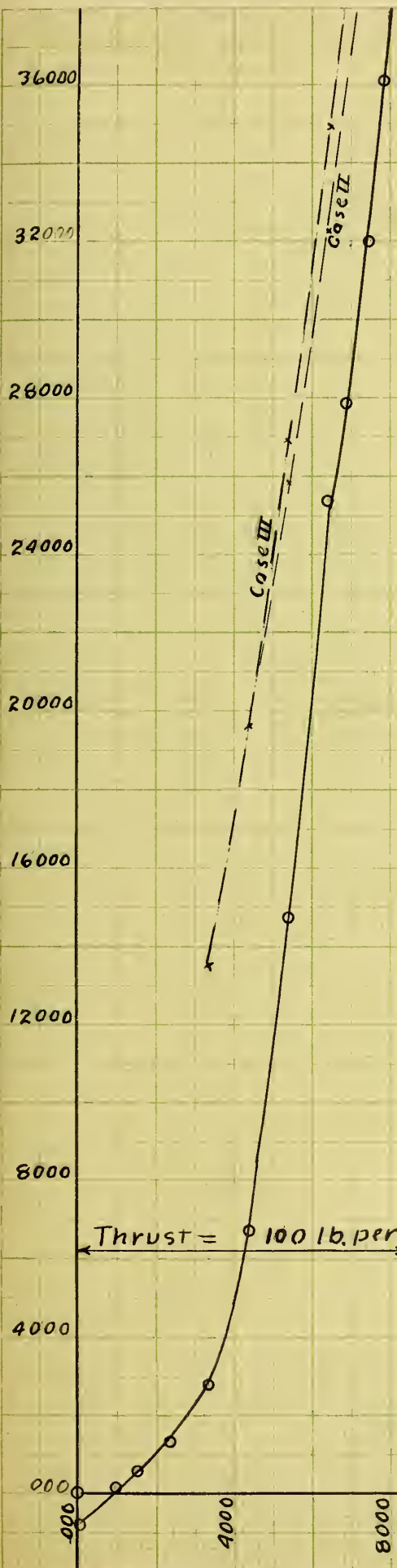
Beam 451.2
 Reinforcement $2 - \frac{1}{2} \phi$ Bottom
 $p = .447\%$ $r = .89$ $n = 12.5$
 —○—○— Exp. Curve *** Theor. Curve
 Failure Due to Tension in Steel

Thrust = 100 lb. per sq. in. of Section

Equivalent $\frac{1}{3}$ Pt. Load in Pounds

Diagram 43

Stress in Steel lb. per sq. in. Tension



Beam 451.2

Reinforcement $2-\frac{1}{2}\Phi$ Bottom $p = 4.47\%$ $r = .89$ $n = 12.5$

○—○ Exp. Curve *** Theor. Curve

Failure Due to Tension in Steel

Thrust = 100 lb. per sq. in. of Section

Equivalent $\frac{1}{3}$ Pt. Load in Pounds

Diagram 44.

Stress in Concrete lb. per sq. in.

Depth of Neutral Axis

Load Released and Reapplied
Stresses Based on Net Deformation

Beam 451.3
Reinforcement 2-1/2" Ø Bottom
P = 45% r = .90 n = 12.5
Exp. Curve
Failure Due to Compression

Thrust in lb. per sq. in. of Section

300 600 900

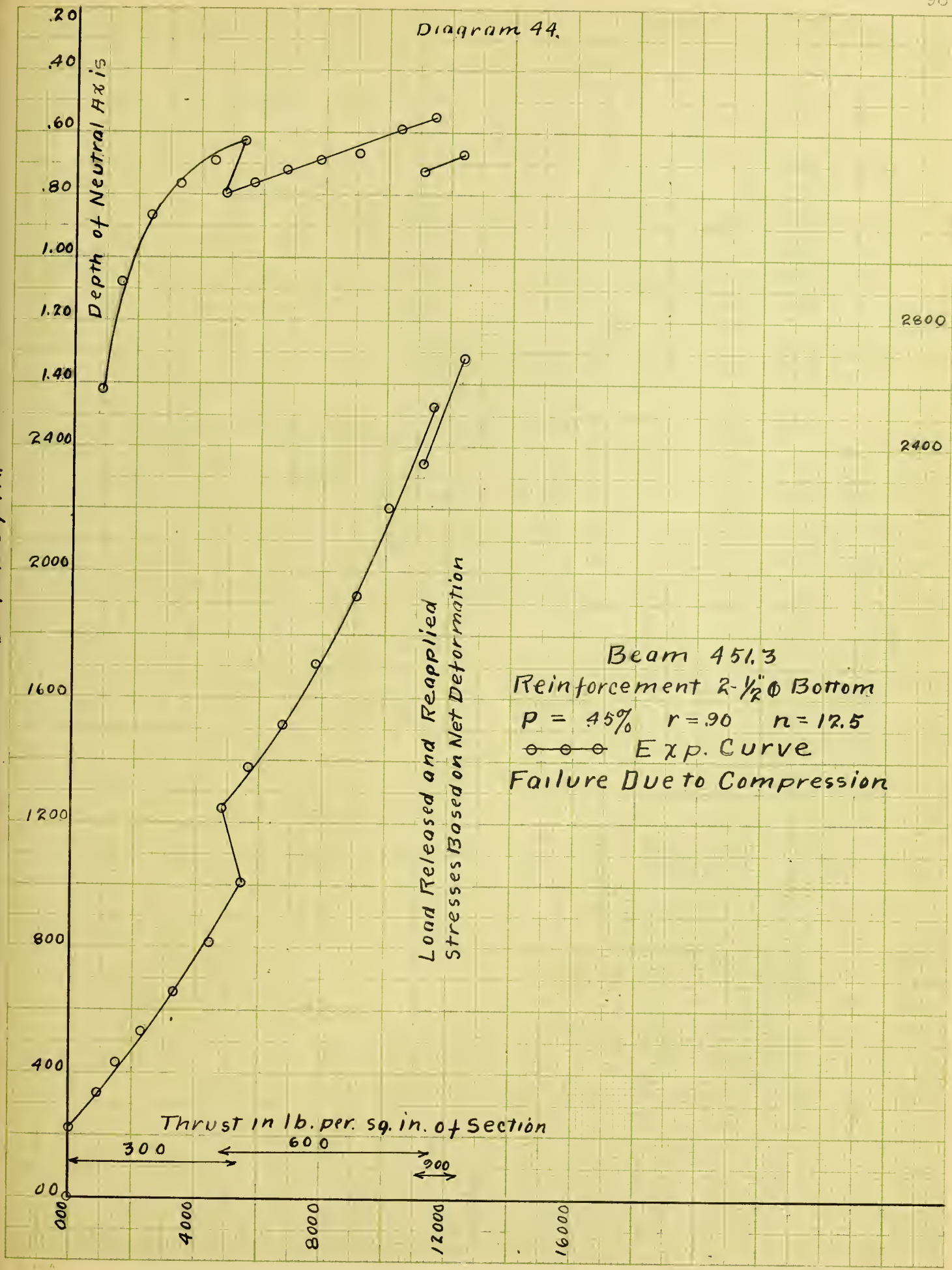
Equivalent 1/3 Pt. Load in Pounds

2800

2400

0.20
0.40
0.60
0.80
1.00
1.20
1.40
2400
2000
1600
1200
800
400
00

0000 4000 8000 12000 16000



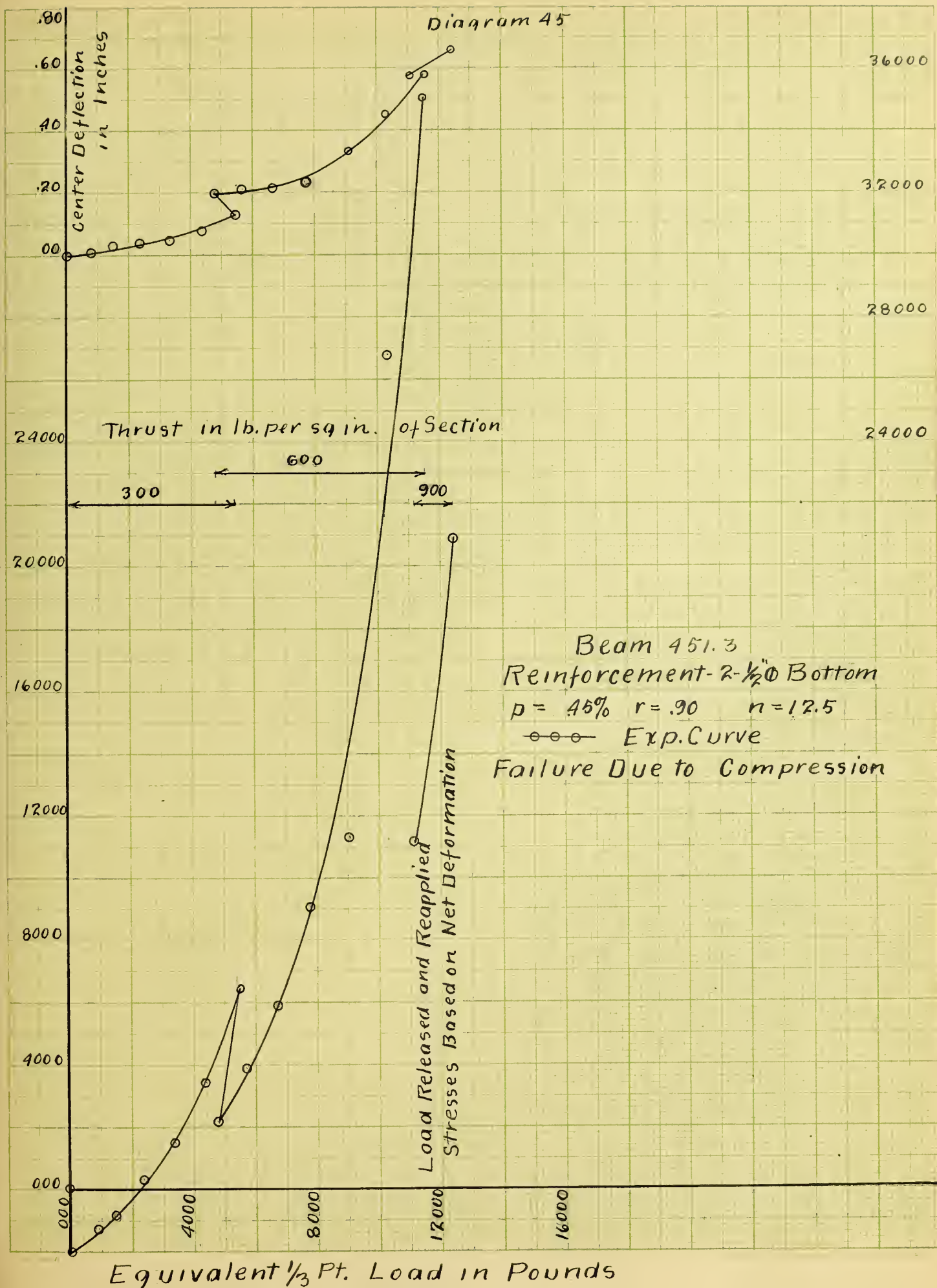
Stress in Steel $\text{lb. per sq. in. Tension}$ 

Diagram 46

Stress in Concrete Lb. per sq. in.

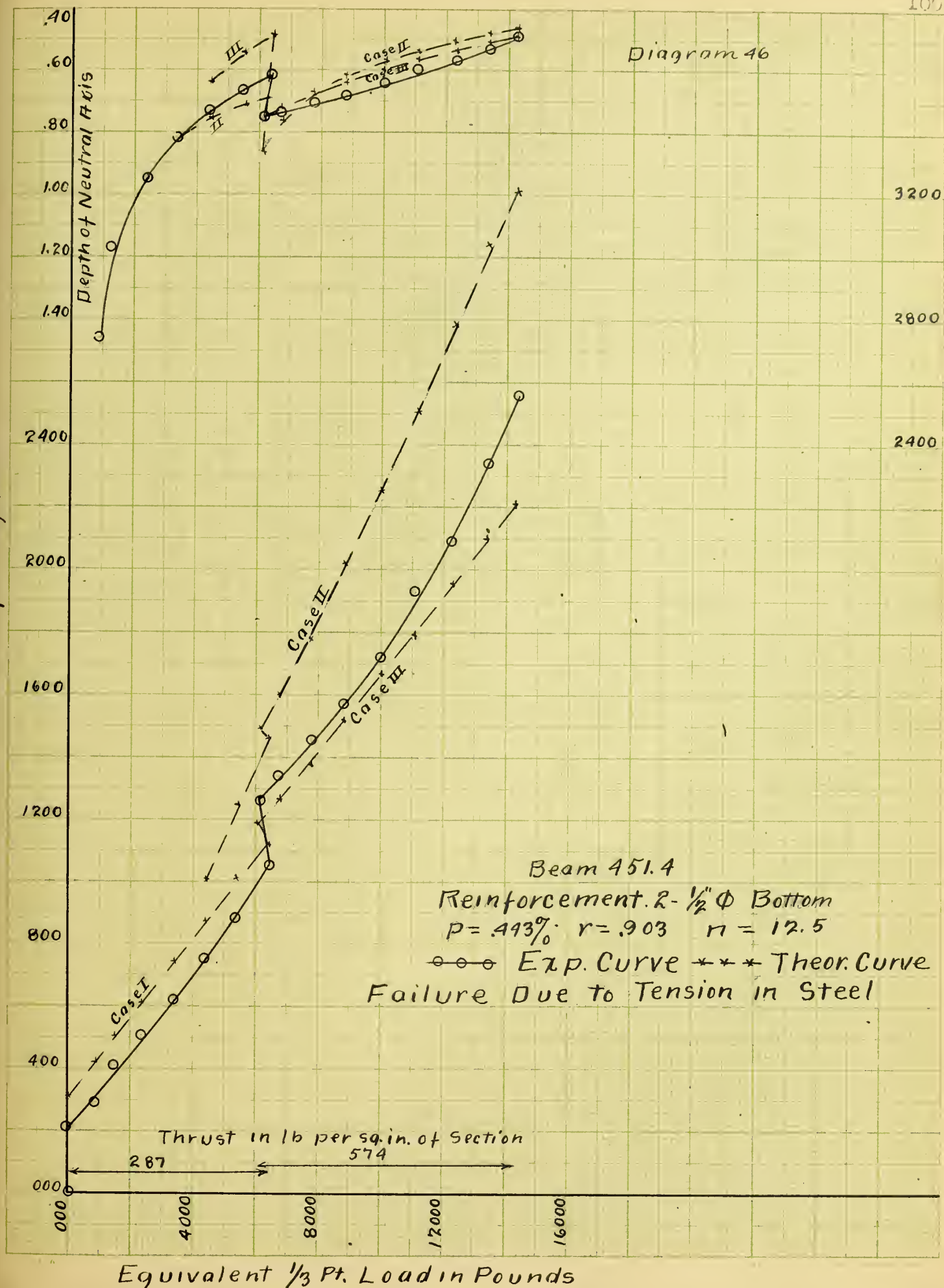
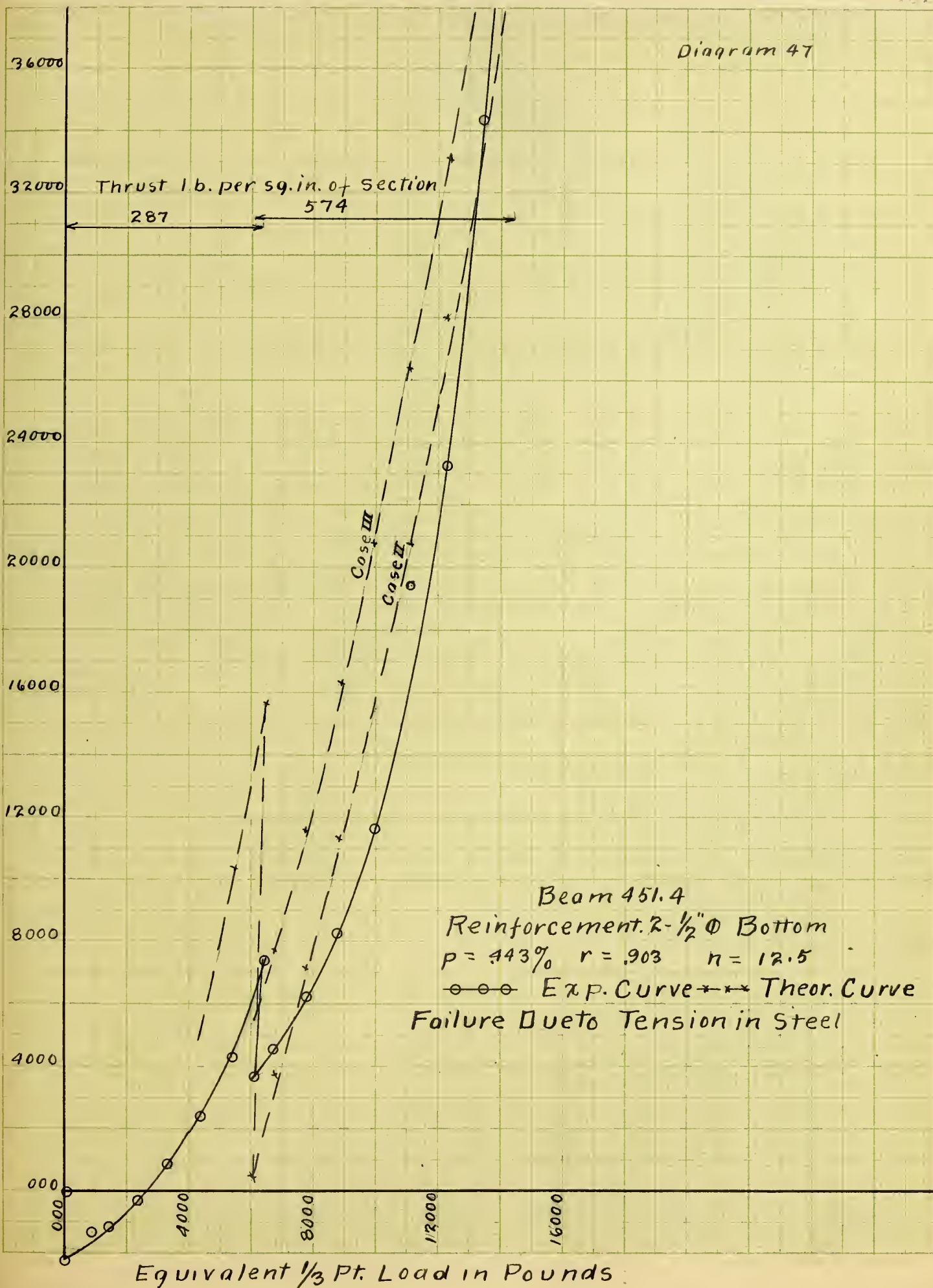


Diagram 47

Stress in Steel Lb per sq. in. Tension



Stress in Concrete lb. per sq. in.

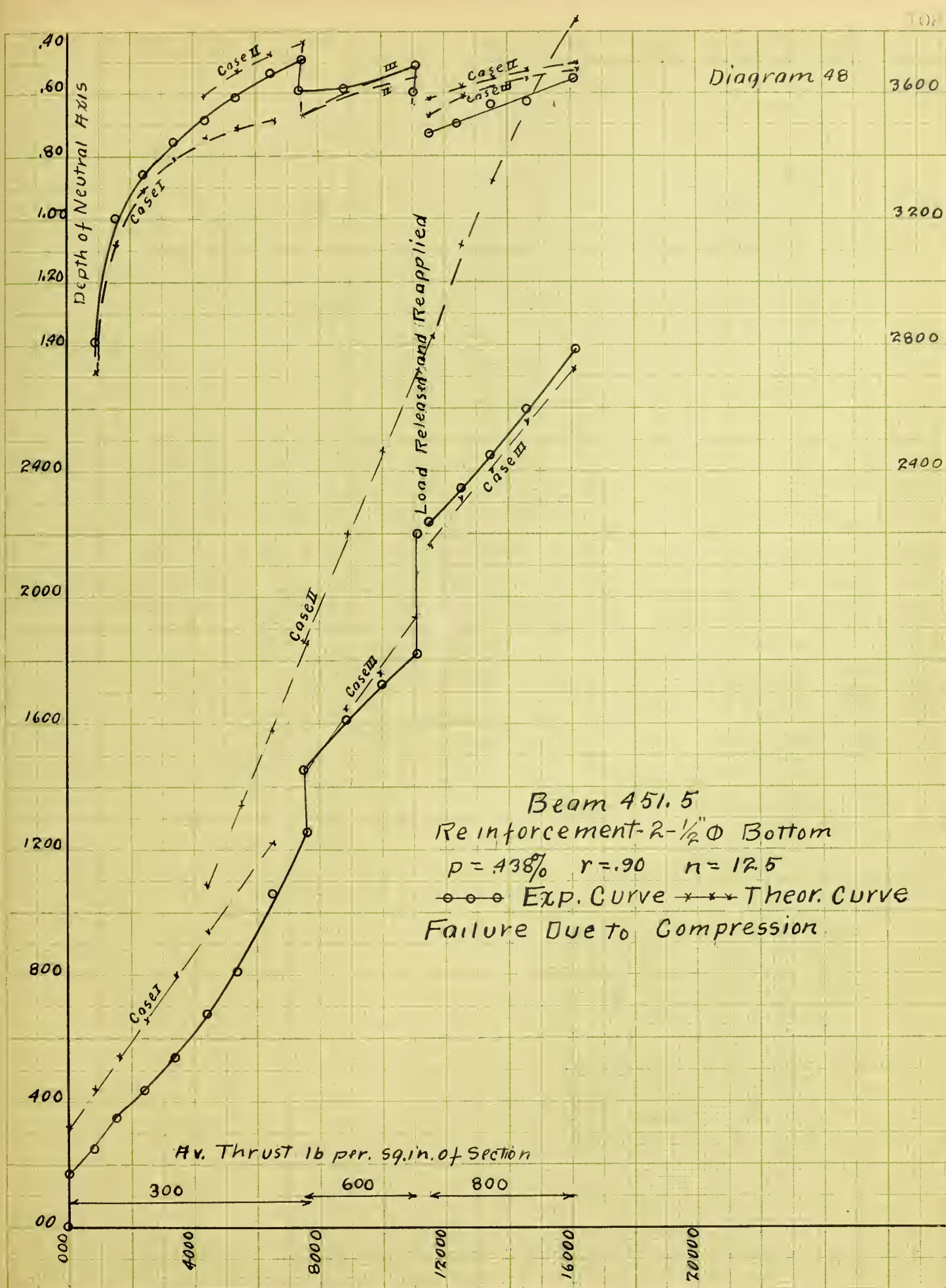


Diagram 49

Stress in Steel, Lb. per sq. in., Tension

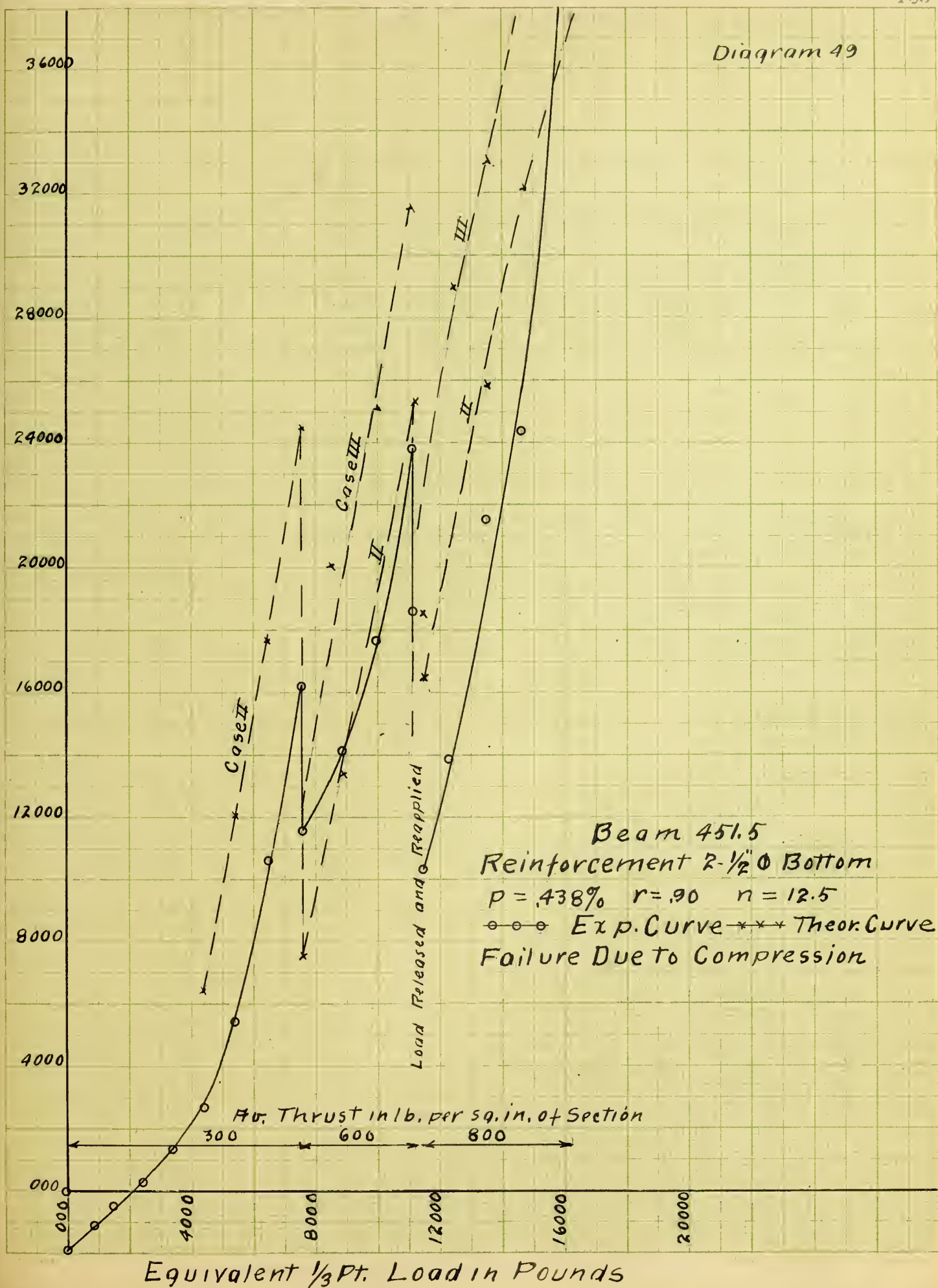


Diagram 50

Stress in Concrete lb. per sq. in.

Depth of Neutral Axis

Beam 452.1

Reinforcement 2- $\frac{1}{2}$ " ϕ Bot. 4 $\frac{1}{2}$ " ϕ Top. $p = .428\%$ $p' = .856\%$ $r = .81$ $r' = .132$

Exp. Curve

Failure Due to Compression

at 18450 lb. $\frac{1}{3}$ Pt. Load Thrust = 805 lb./in.

800

600

400

200

00

Thrust = 101 lb. per in. of Section

Thrust = 0

Repetition

Simple Flexure

2000

4000

0000

2000

4000

Equivalent $\frac{1}{3}$ Pt. Load in Pounds

Diagram 51

Stress in Steel Lb. per sq. in. Tension

Center Deflection
in Inches

.00

.10

.20

Repetition

Simple Flexure

Beam 452.1

Reinforcement 2- $\frac{1}{2}$ " Φ Bot. 4- $\frac{1}{2}$ " Φ Top. $p = .428\%$ $p' = .856\%$ $r = .81$ $r' = .132$ $n = 12.5$

Exp. Curve

6000

4000

2000

000

Thrust = 101 lb. per sq. in.

Thrust = 0

Repetition

Simple Flexure

Equivalent $\frac{1}{3}$ Pt. Load in Pounds

2000

4000

0000

2000

4000

Diagram 52

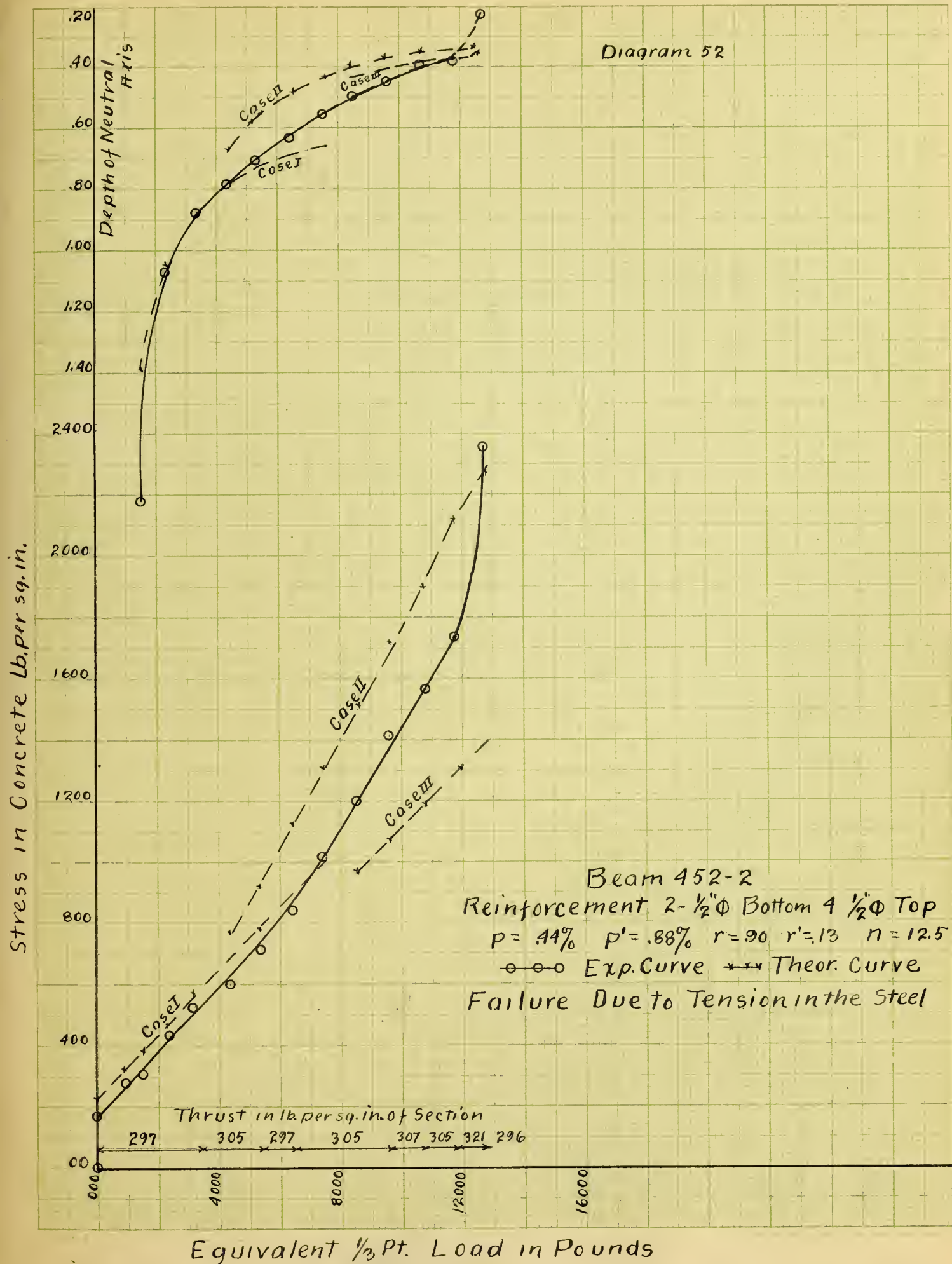


Diagram 53

Stress in Steel, Lb. per sq. in. Tension

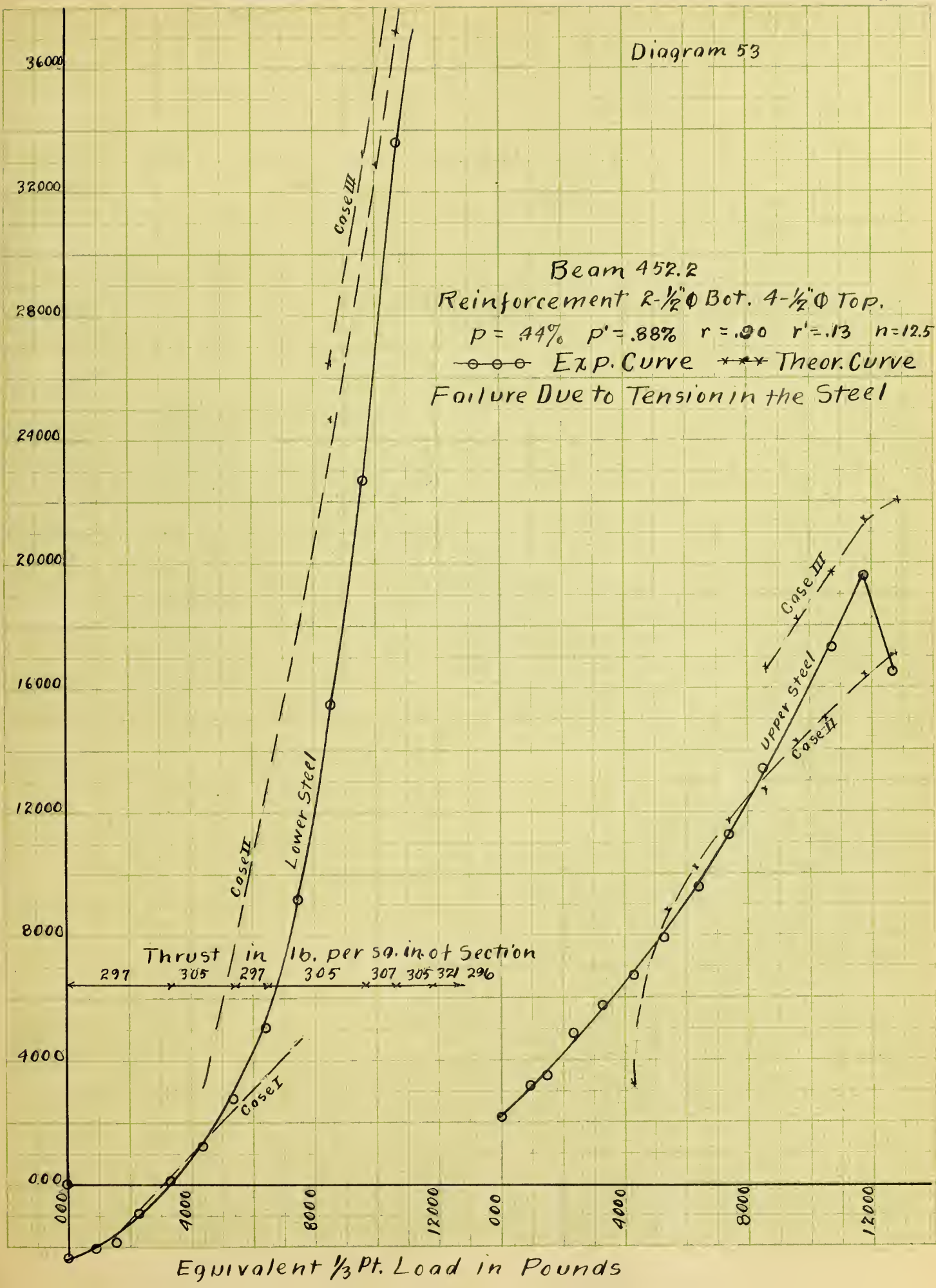


Diagram 54

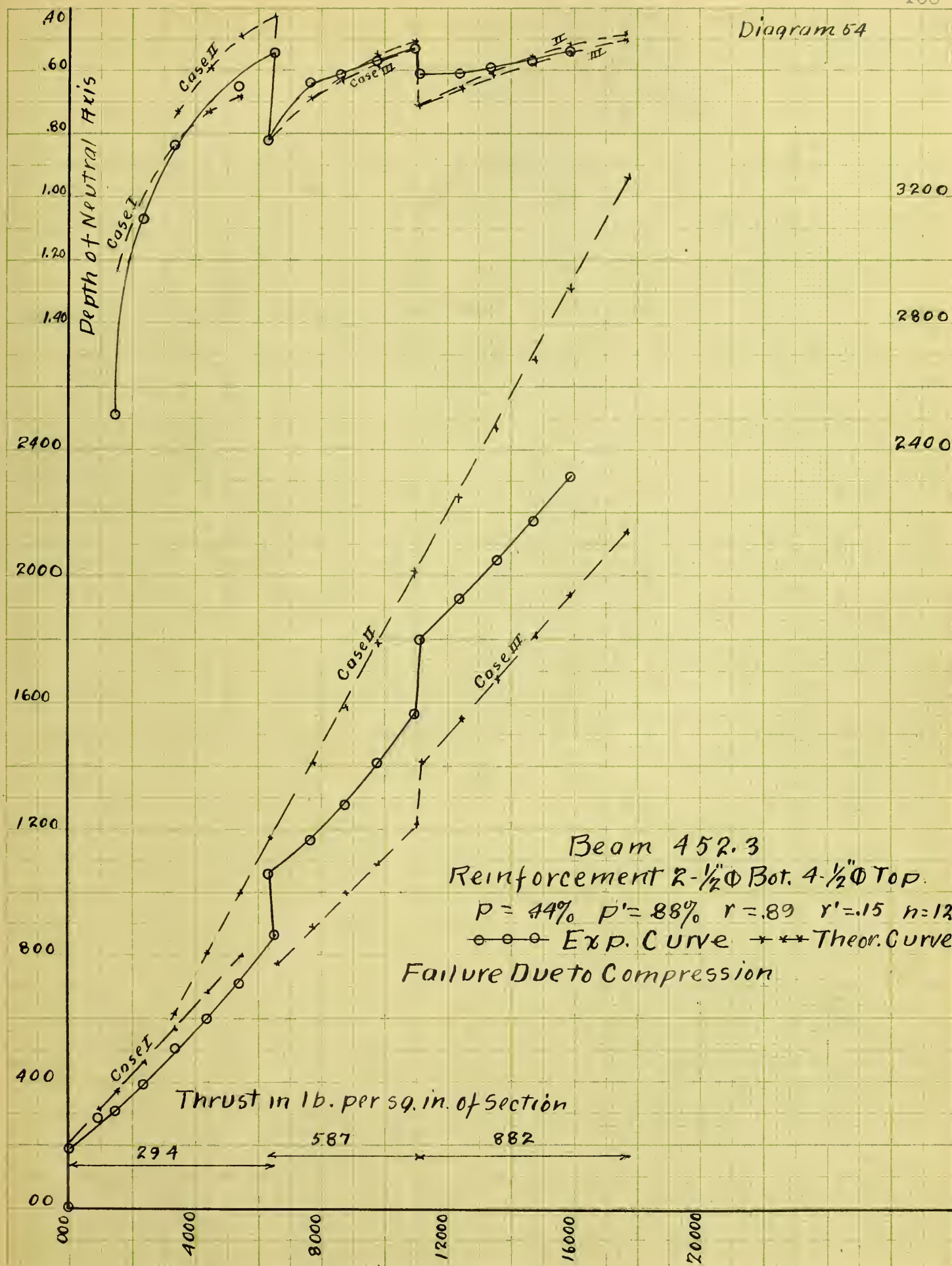
Stress in Concrete lb. per sq. in. 

Diagram 55

Stress in Steel Lb. per sq. in. Tension

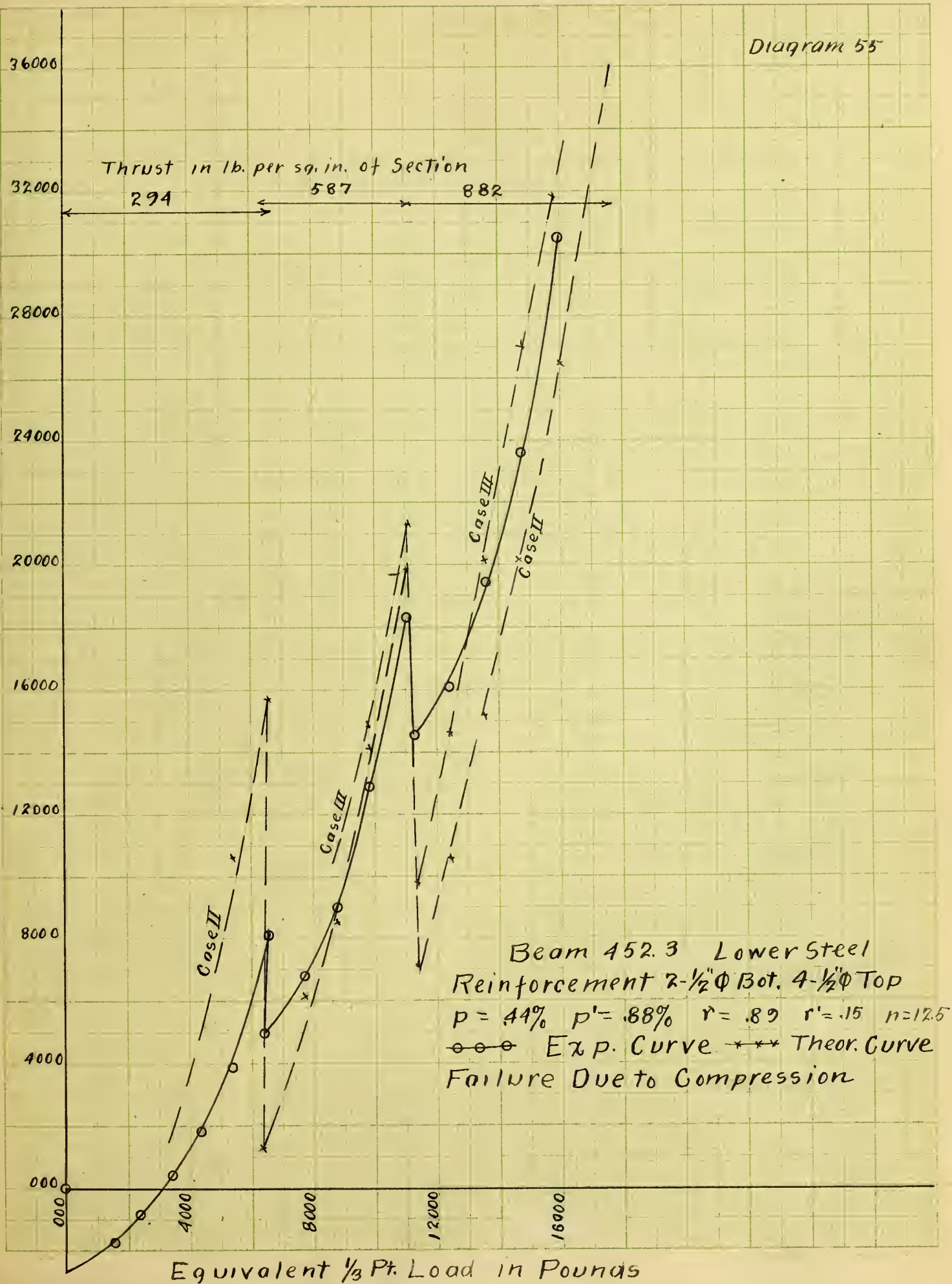
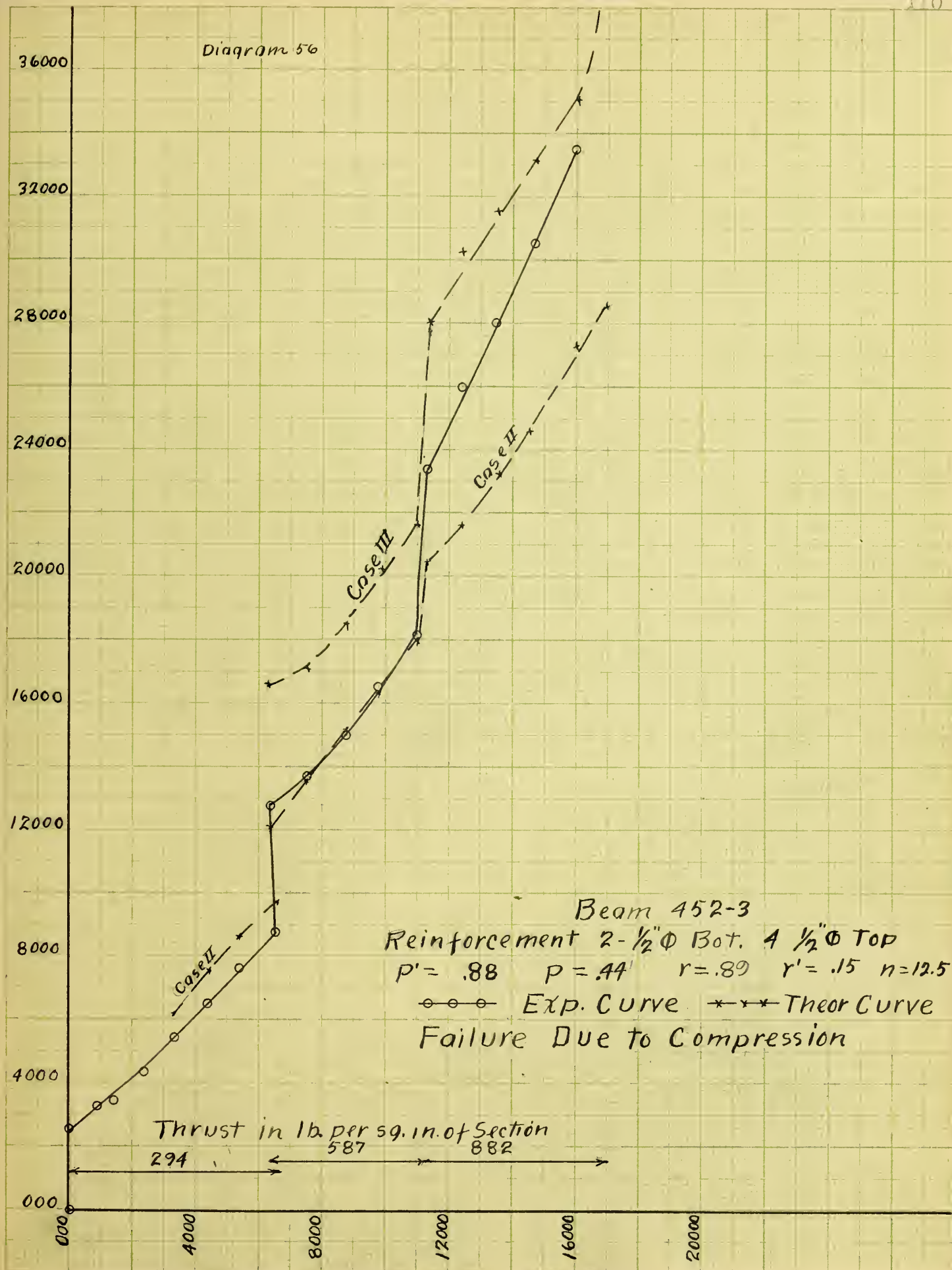


Diagram 56

Compressive Stress in Steel Lb. per sq. in.



Equivalent $\frac{1}{3}$ Pt. Load in Pounds

Diagram 57

Stress in Concrete lb. per sq. in.

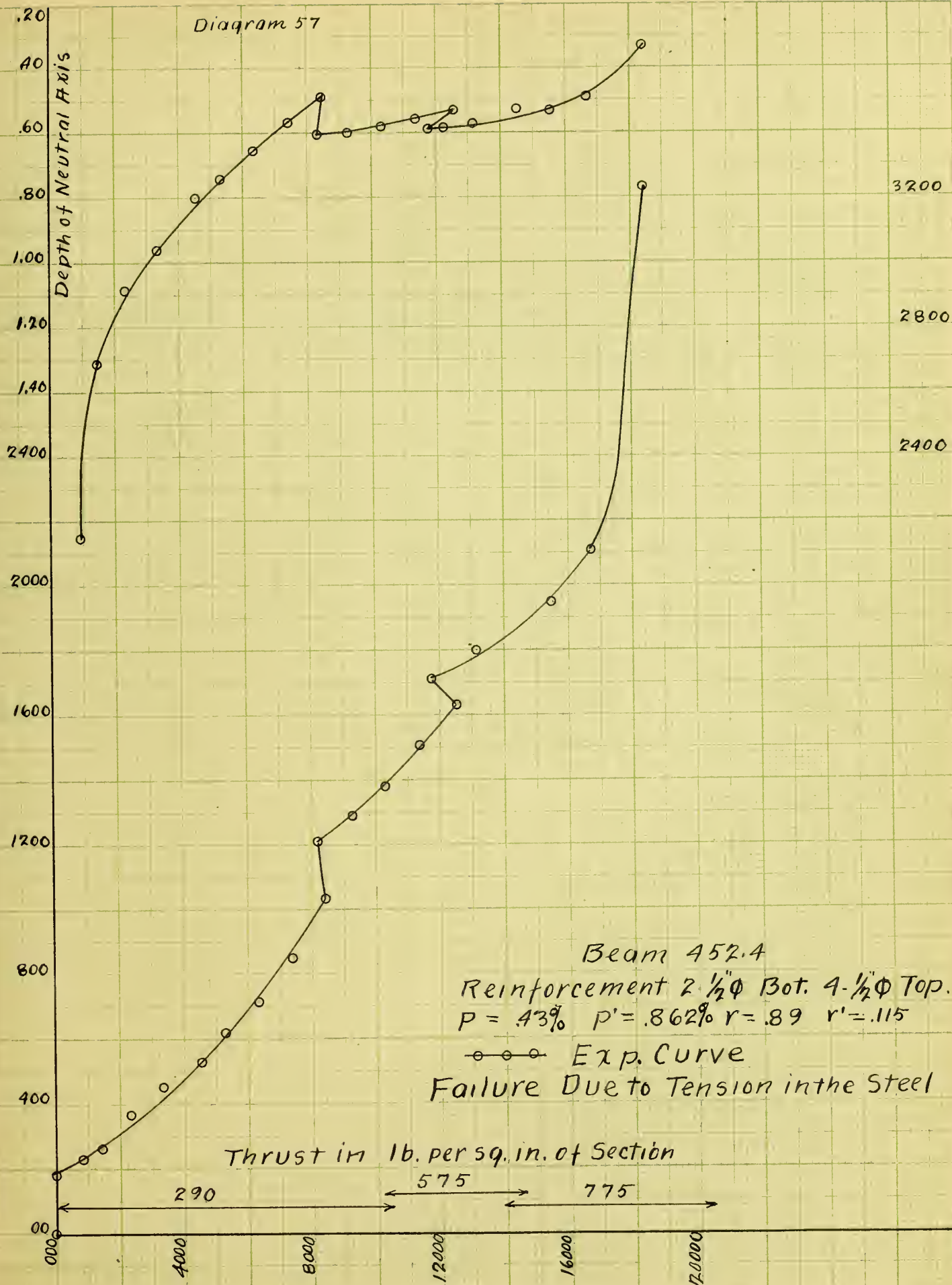
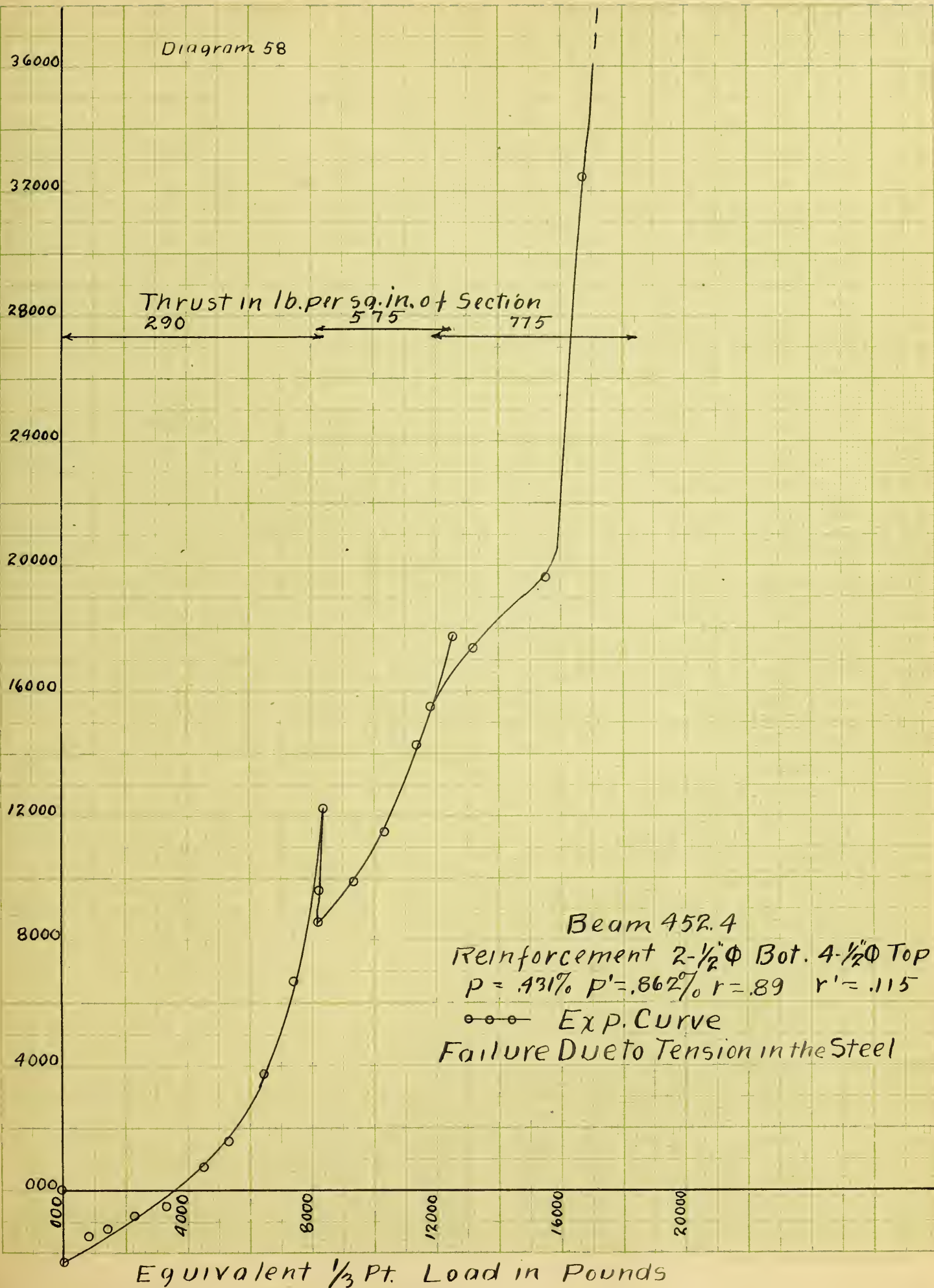
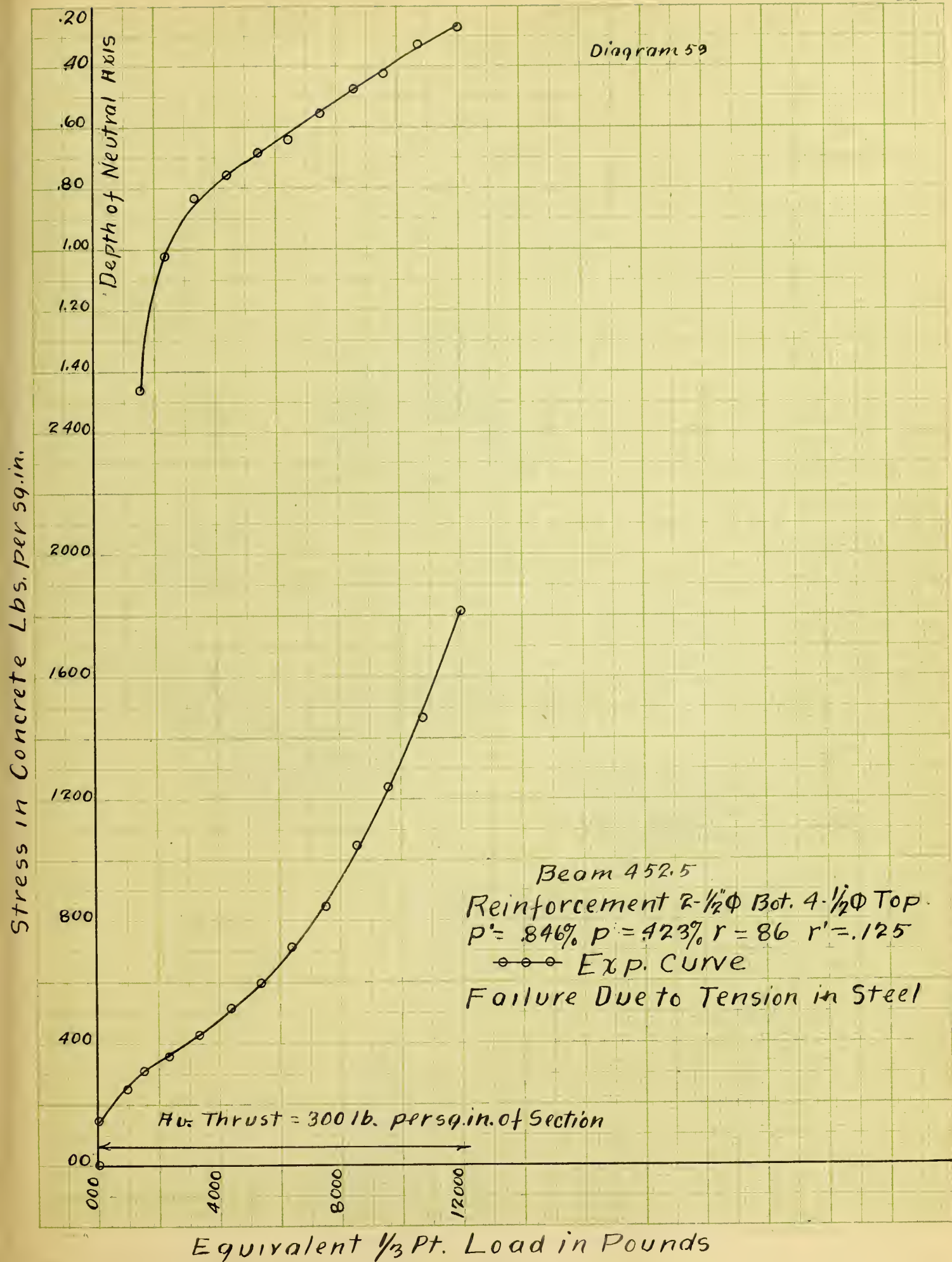


Diagram 58

Stress in Steel Lb. per sq. in. Tension





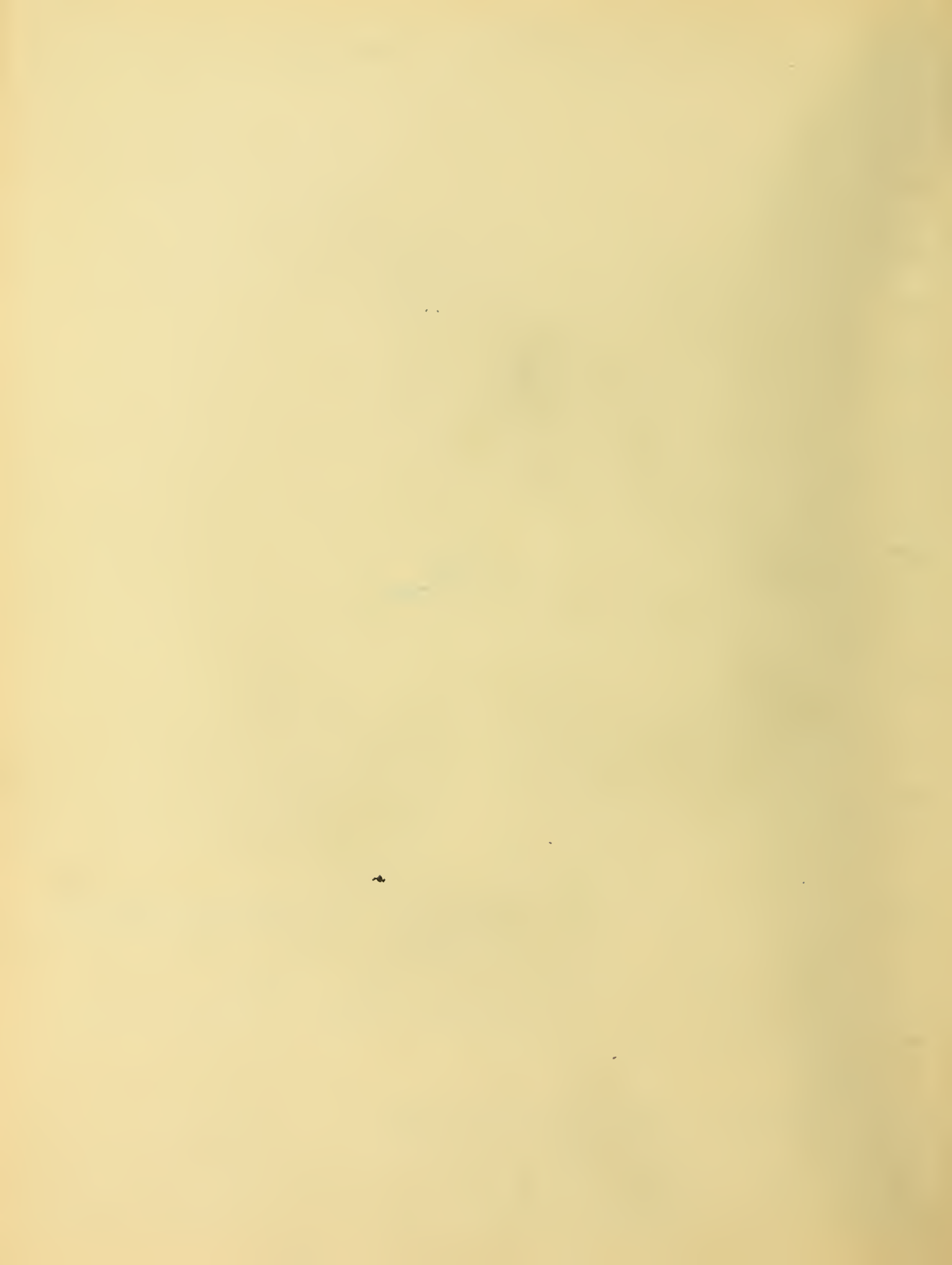
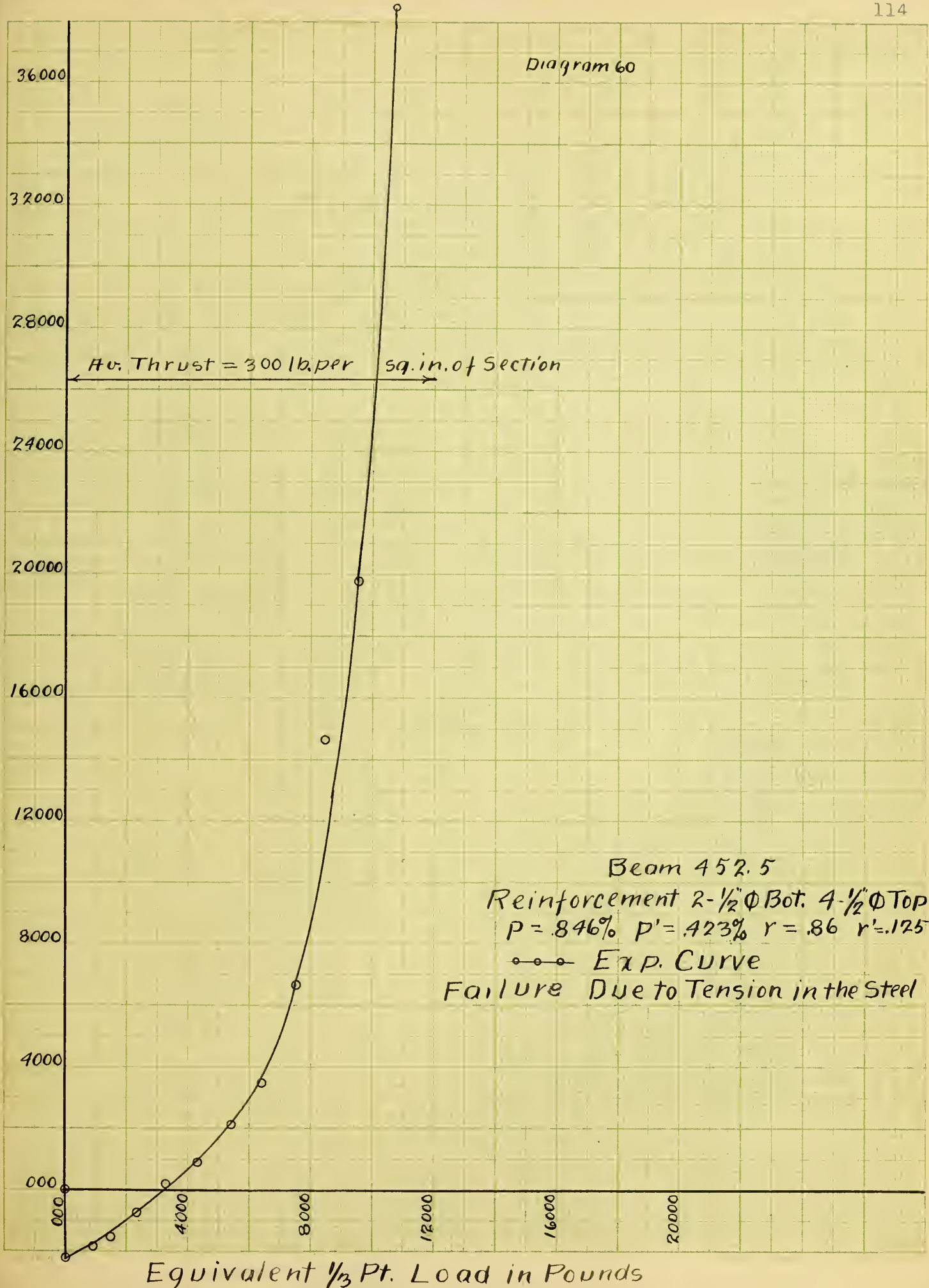


Diagram 60

Stress in Steel Lb. per sq. in. Tension



Stress in Concrete Lb. per sq. in.

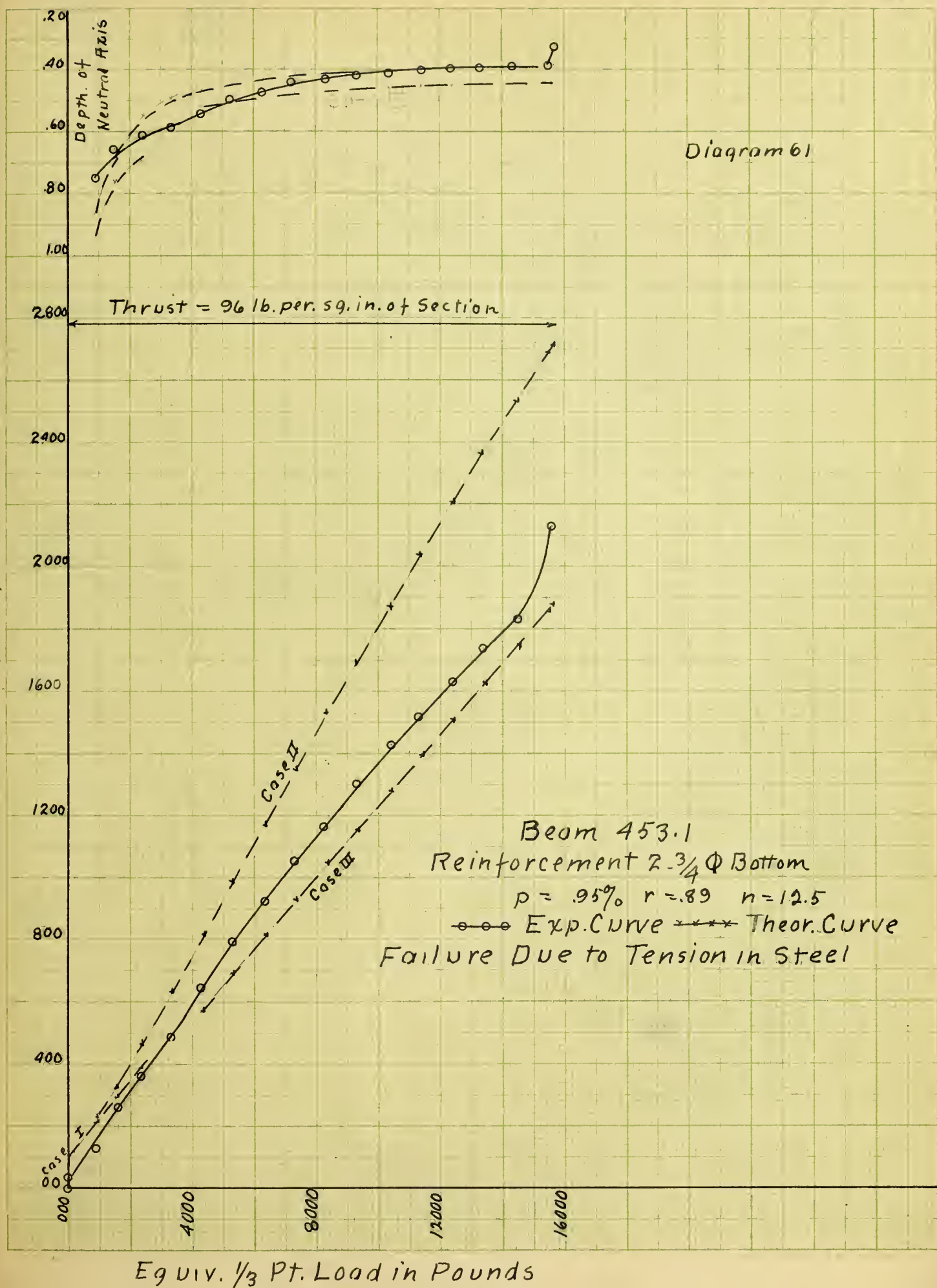
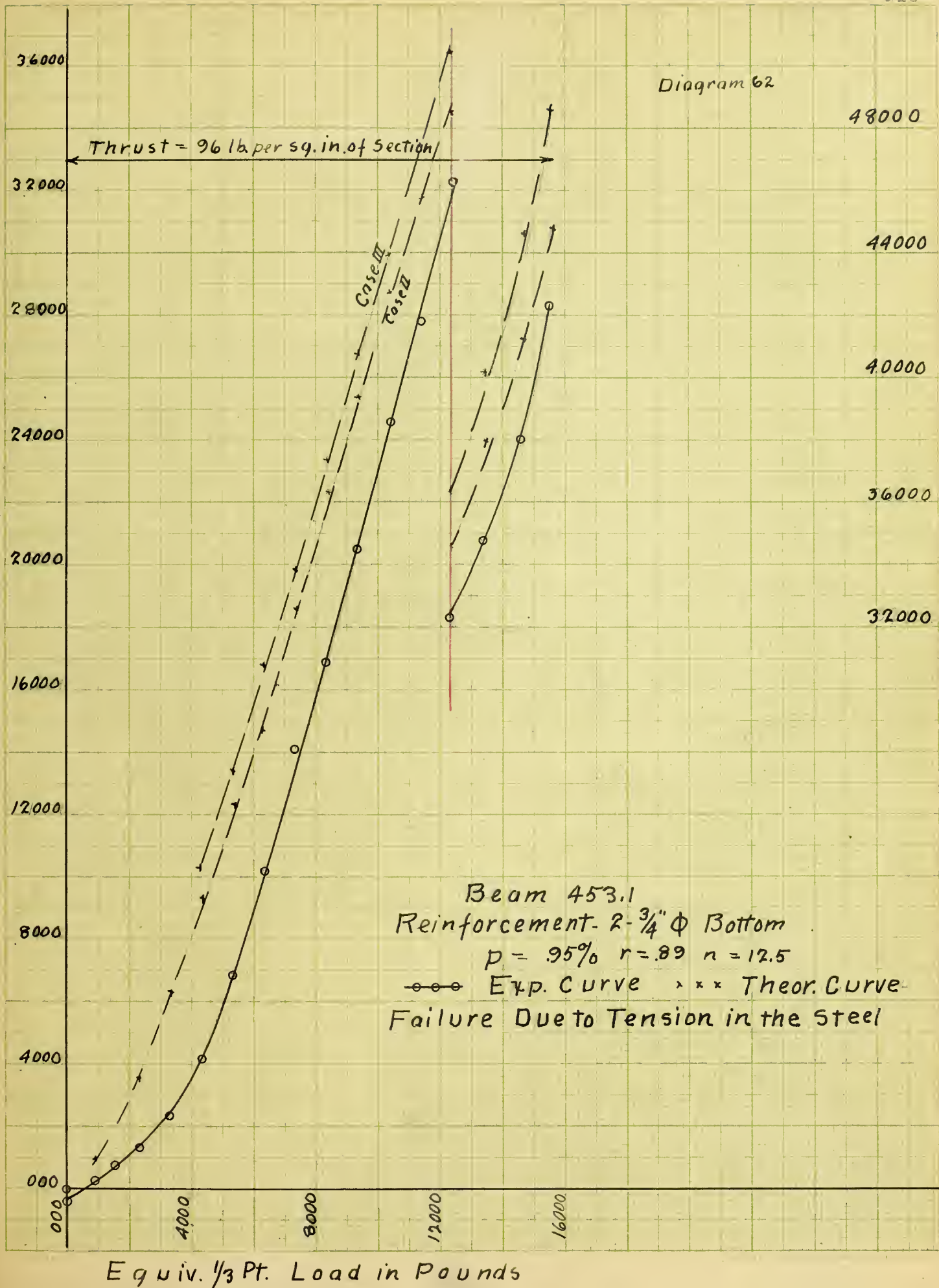
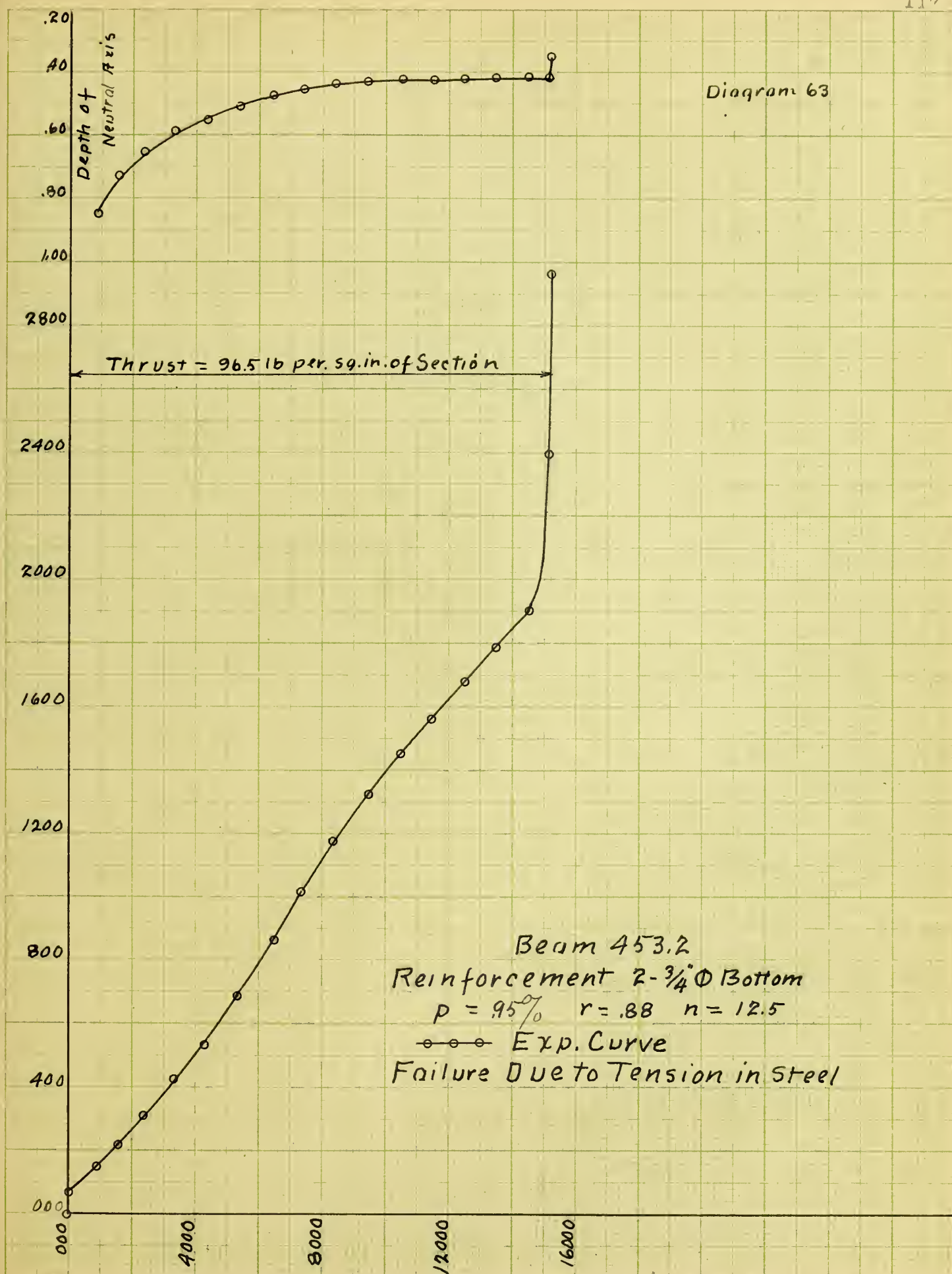


Diagram 62

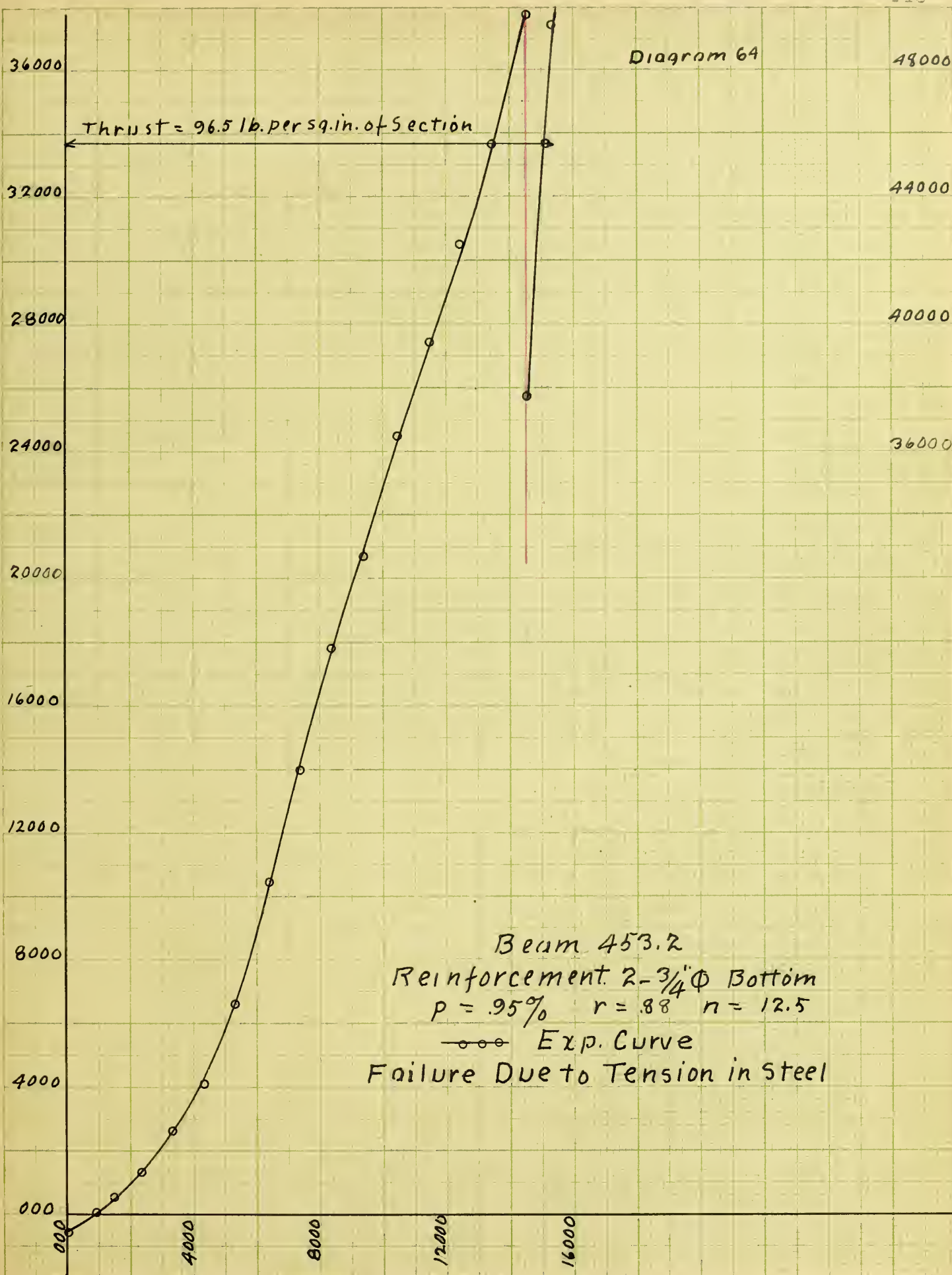
Stress in Steel Lb. per sq. in. Tension



Stress in Concrete Lb. per sq. in.

Equivalent $\frac{1}{3}$ Pt. Load in Pounds

Stress in Steel Lb. per sq. in. Tension.



Stress in Concrete Lb. per sq. in.

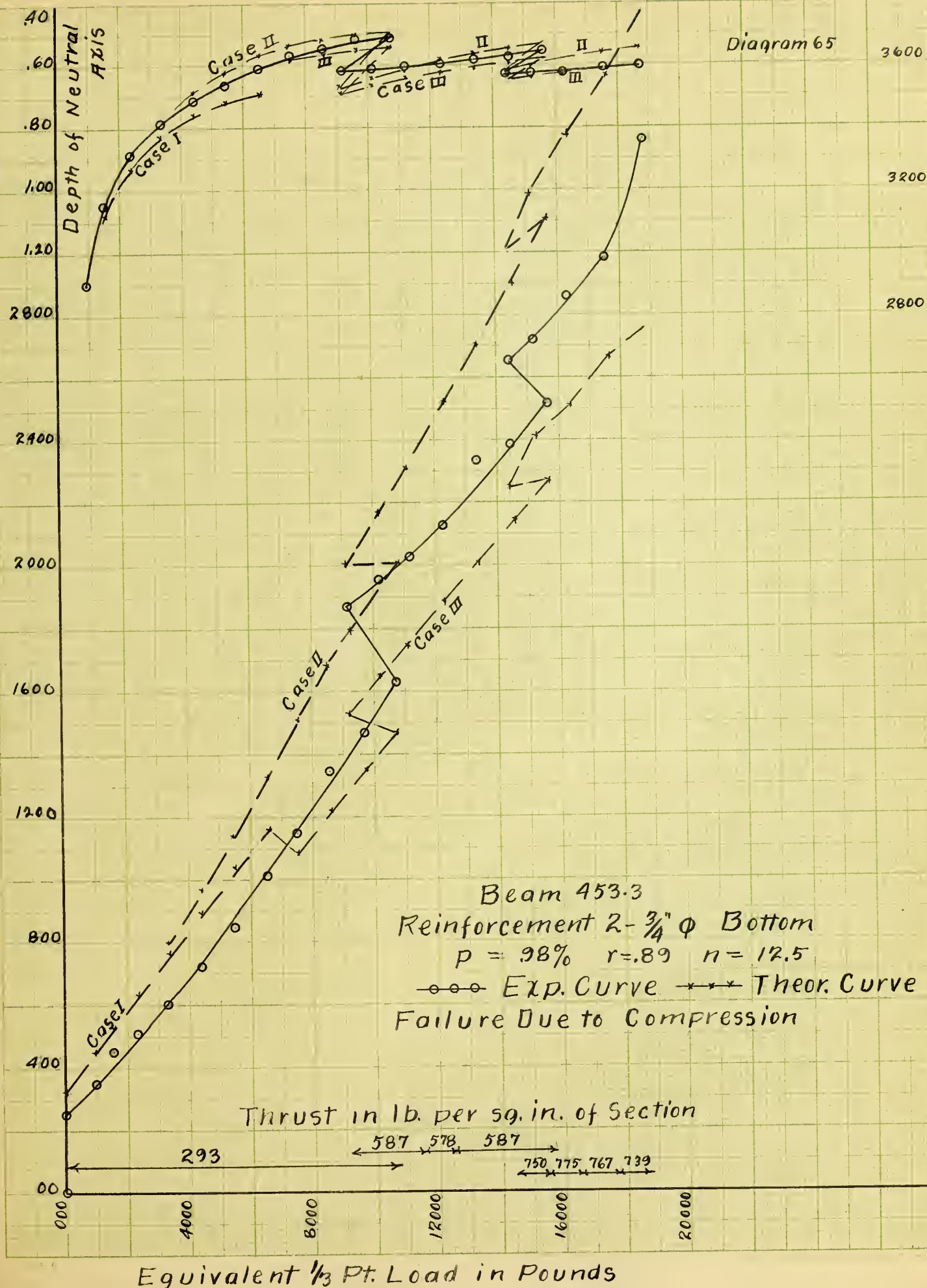


Diagram 66

Stress in Steel Lb. per Sq. In. Tension

36000
32000
28000
24000
20000
16000
12000
8000
4000

Beam 453-3
Reinforcement $2-\frac{3}{4}" \phi$ Bottom
 $p = .985\%$ $n = 12.5$ $r = .89$
 ○—○—○ Exp. Curve
 ——* Theor. Curve
 Failure due to Compression

Thrust in Lb. per Sq. In. of Section.

293 587 578 587 750 716 767 739

Equivalent $\frac{1}{3}$ Pt. Load in Pounds

4000 8000 12000 16000 20000

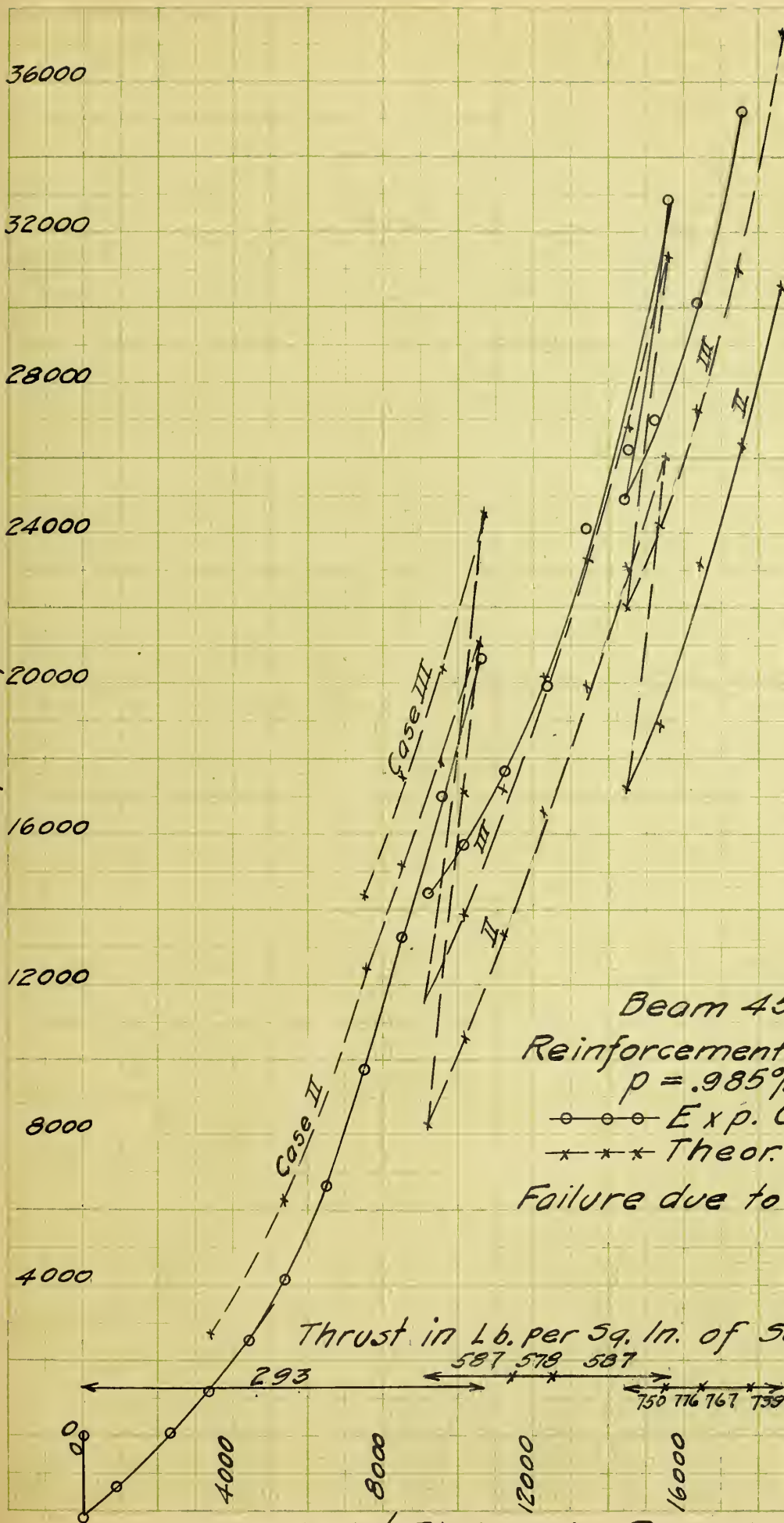
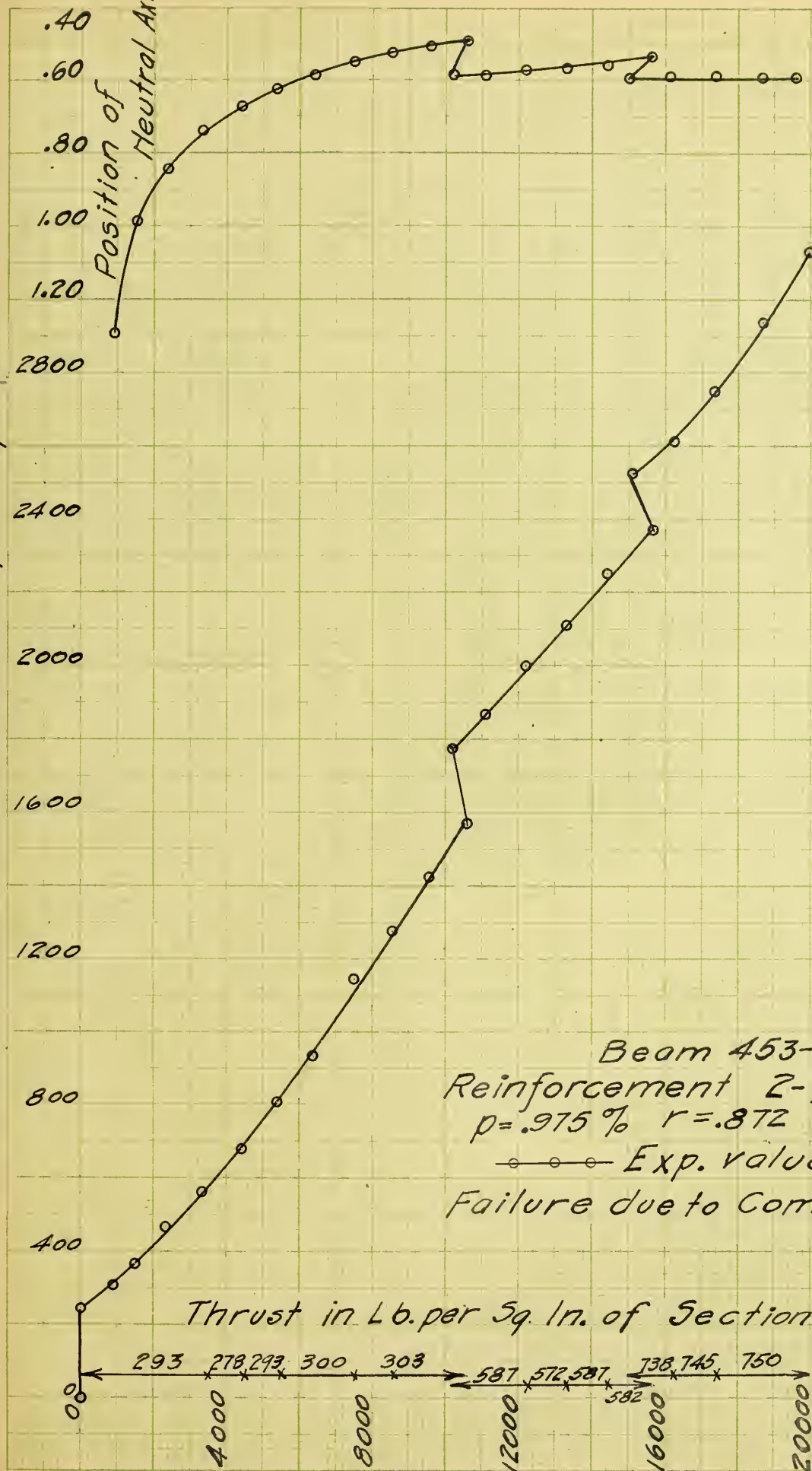


Diagram 67

Stress in Concrete Lb. per Sq. In.

Position of Neutral Axis.

Equivalent $\frac{1}{3}$ Point Load in Pounds

Stress in Steel Lb. per Sq. In. Tension

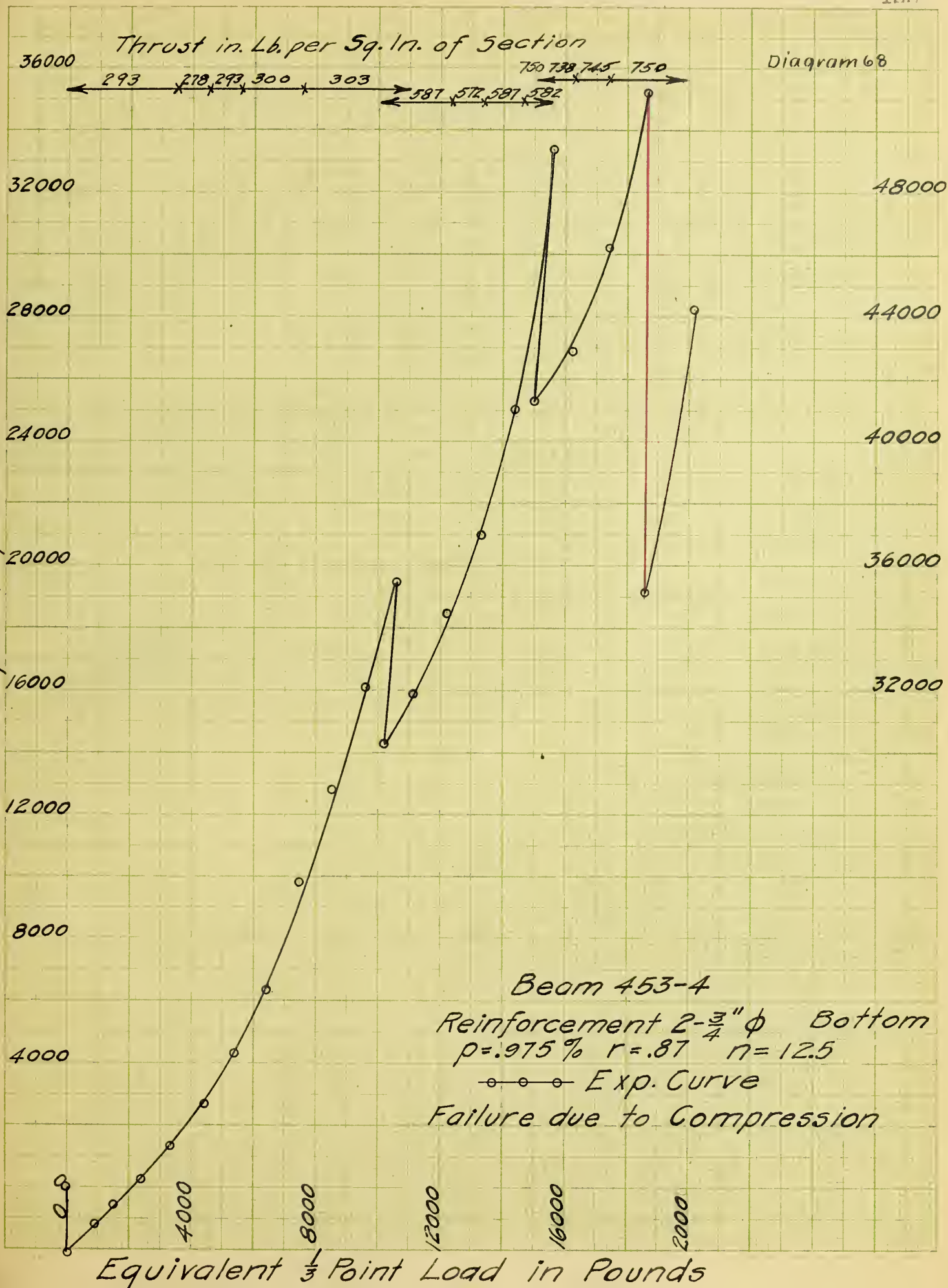
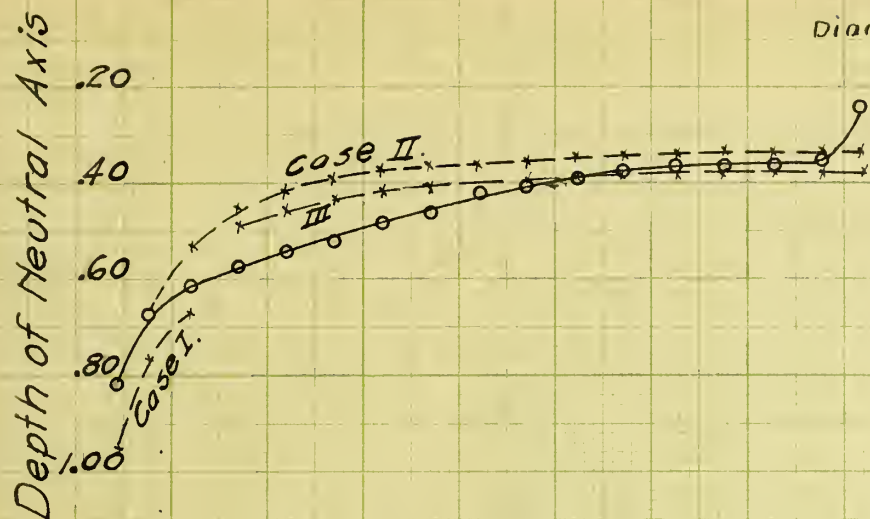


Diagram 69



Compressive Stress in Concrete Lb. per Sq. In.

2400

2000

1600

1200

800

400

End Load = $100\frac{1}{4}$ " of section

Case I.

Case II.

Case III.

Beam 454-1

Reinforcement $2-3"$ ϕ Top and Bottom $\rho = \rho' = .975\%$ $n = 12.5$

—○—○—○— Exp. Curve

—×××— Theor. Curve

Failure by Tension in Steel

Equivalent $\frac{1}{3}$ Point Load in Pounds

4000

8000

12000

16000

20000

Diagram 70

Stress in Steel $\text{lb. per sq. in. Tension}$

36000

32000

28000

24000

20000

16000

12000

8000

4000

48000

44000

40000

36000

32000

28000

Thrust = $100 \frac{\text{lb}}{\text{sq. in.}}$ of Section

Beam 454-1

Reinforcement $2-\frac{3}{4}" \phi$ Top and Bottom $p = p' = .975\%$ $n = 12.5$

○—○—○ Exp. Curve

——* Theor. Curve

Failure by Tension in Steel

Equivalent $\frac{1}{3}$ Point Load in Pounds.

4000

8000

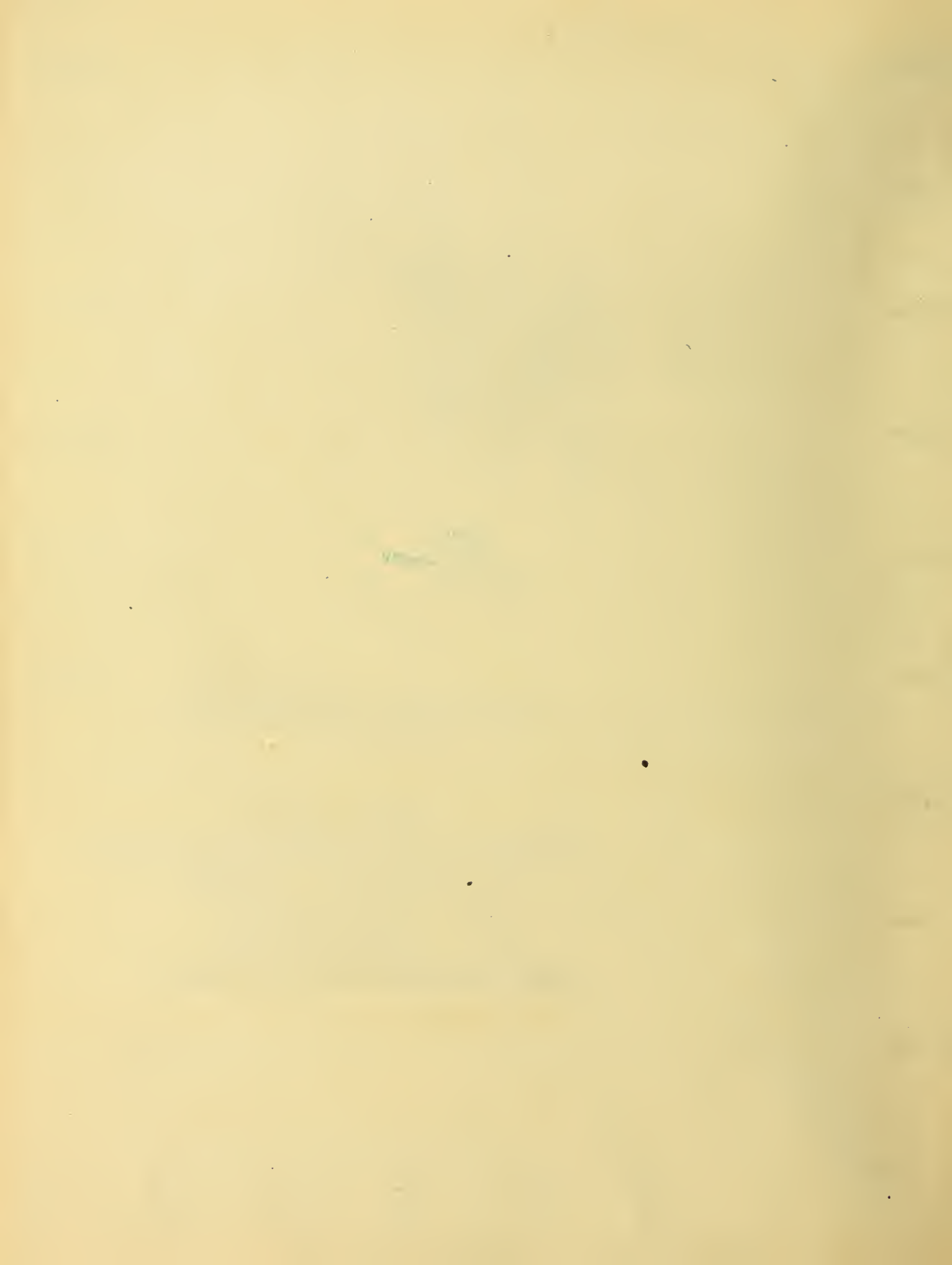
12000

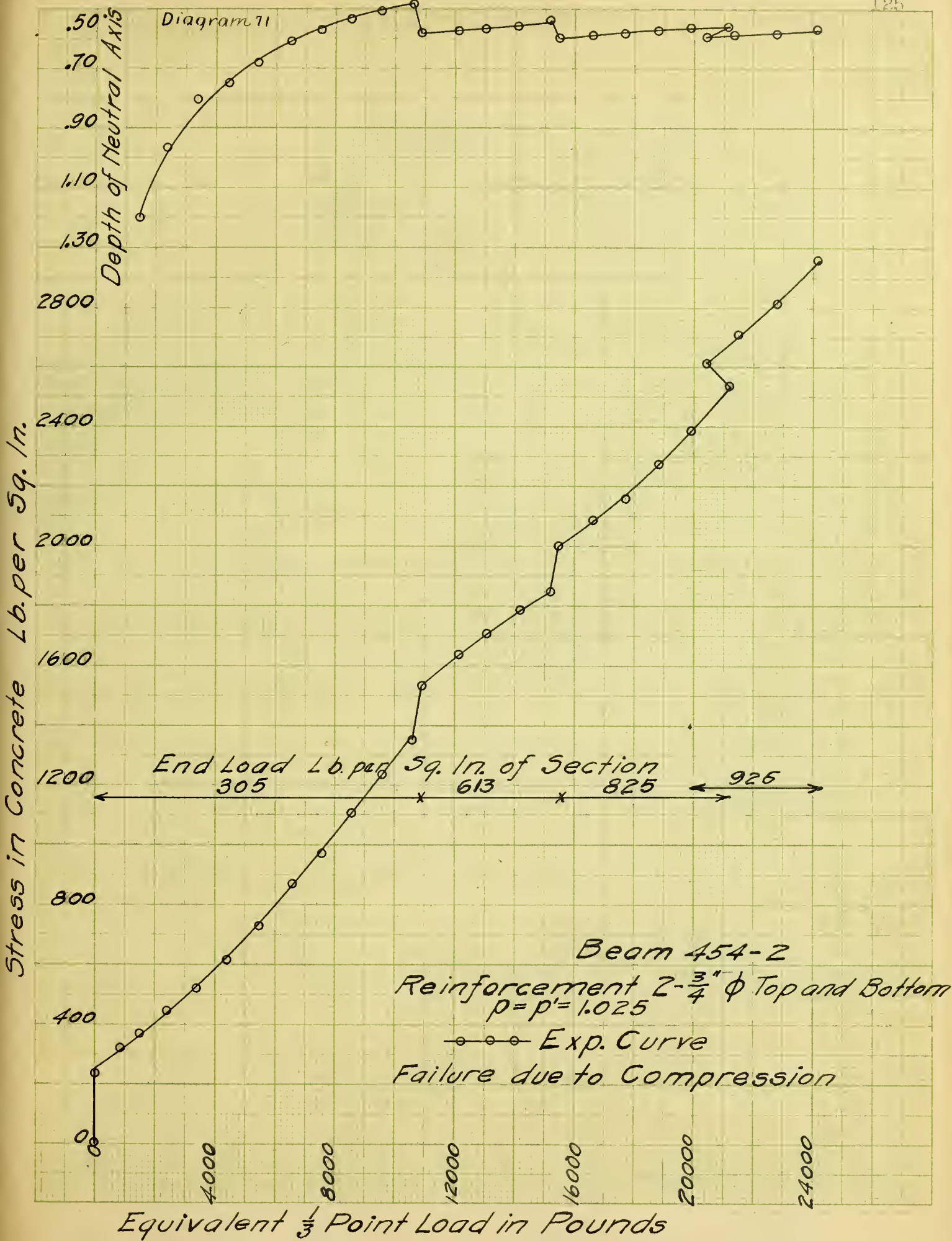
16000

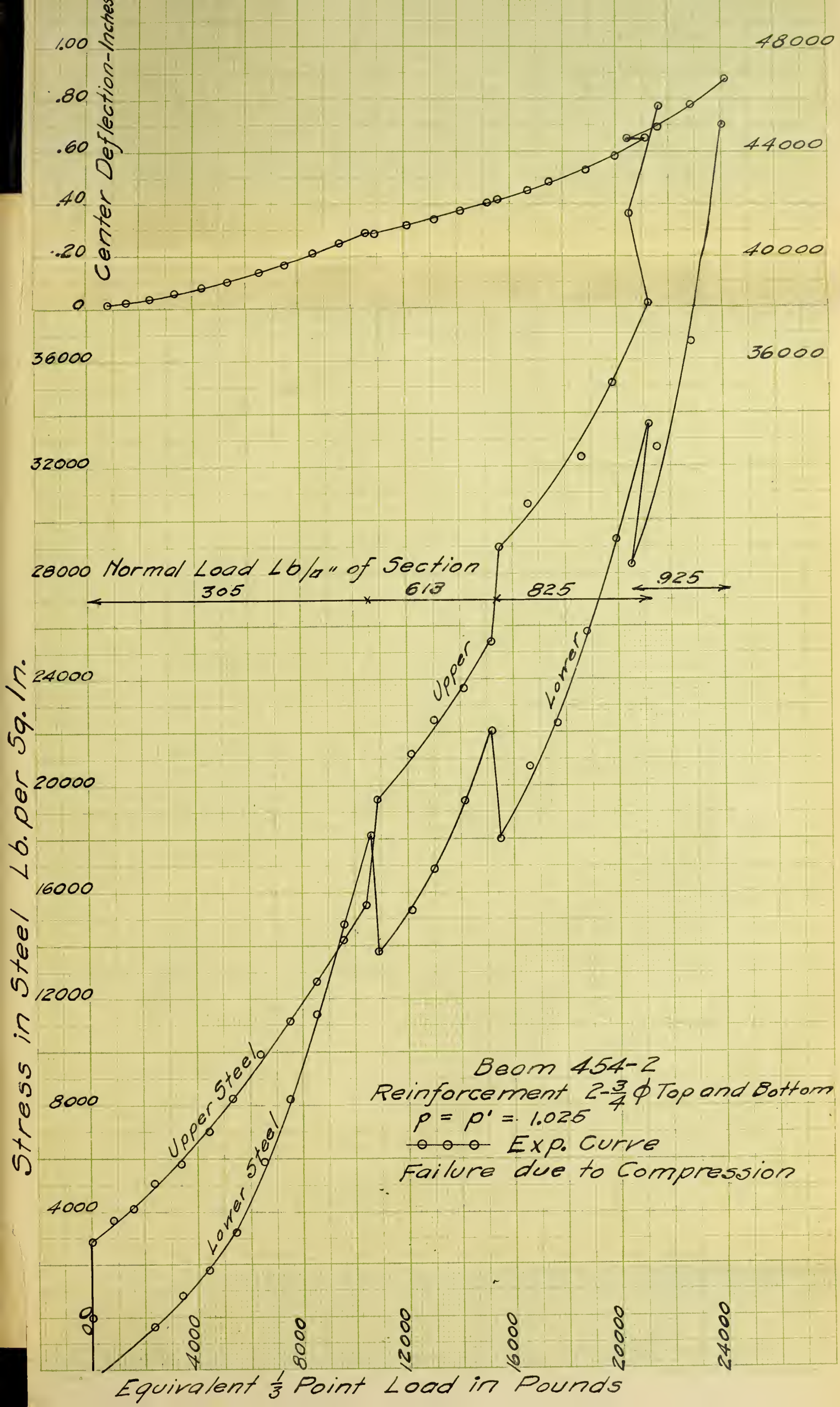
20000

24000

28000







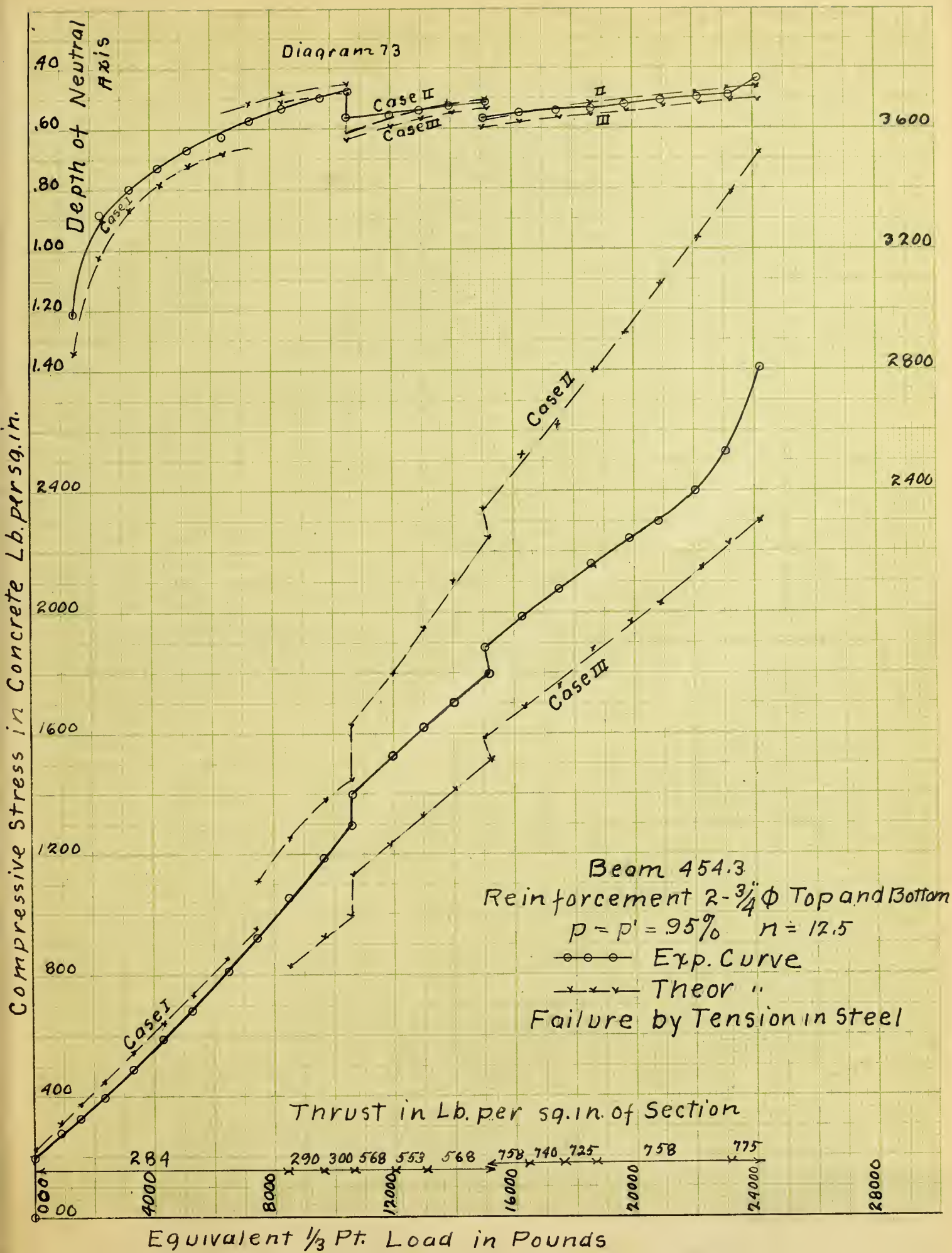
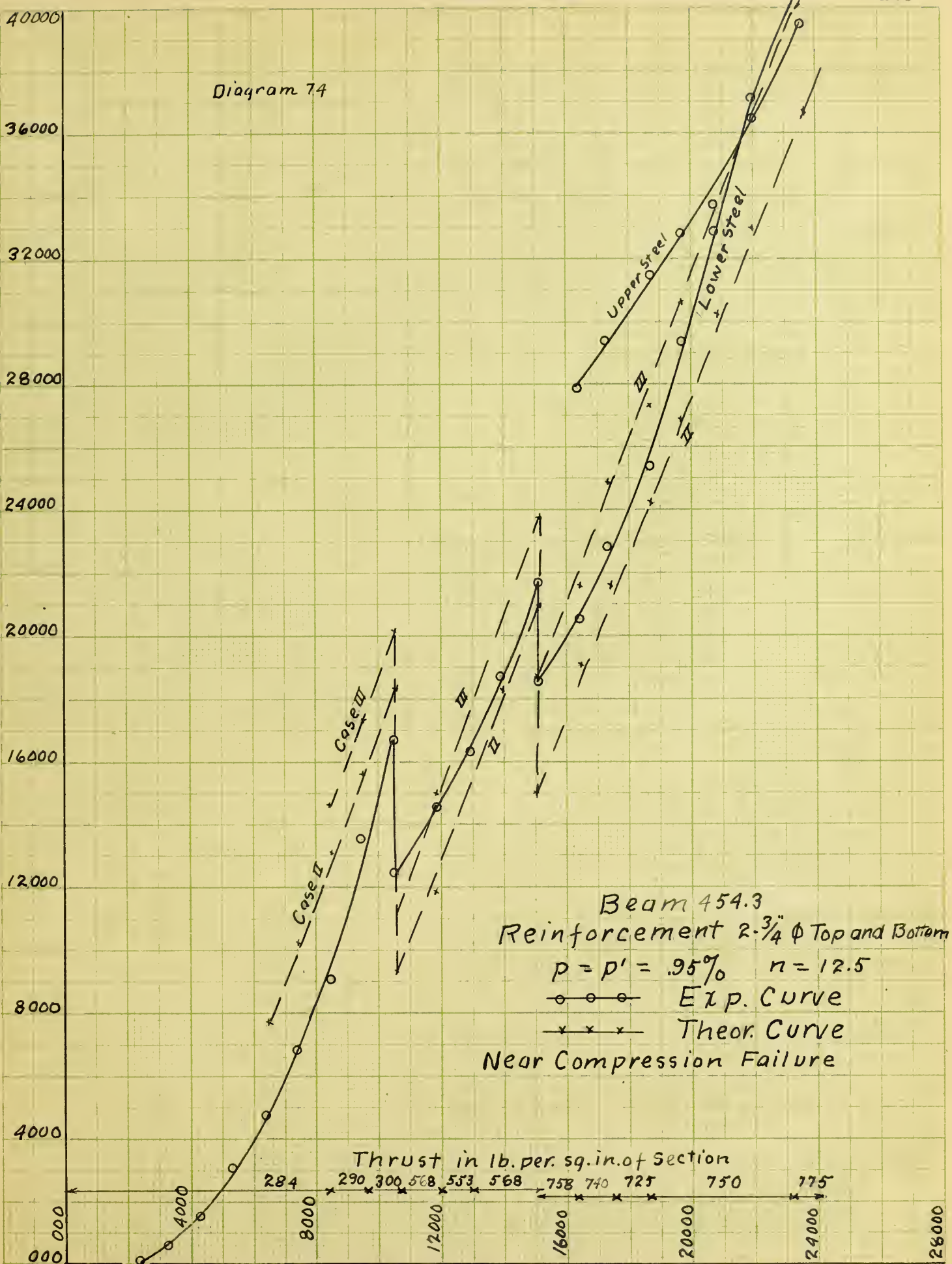


Diagram 74

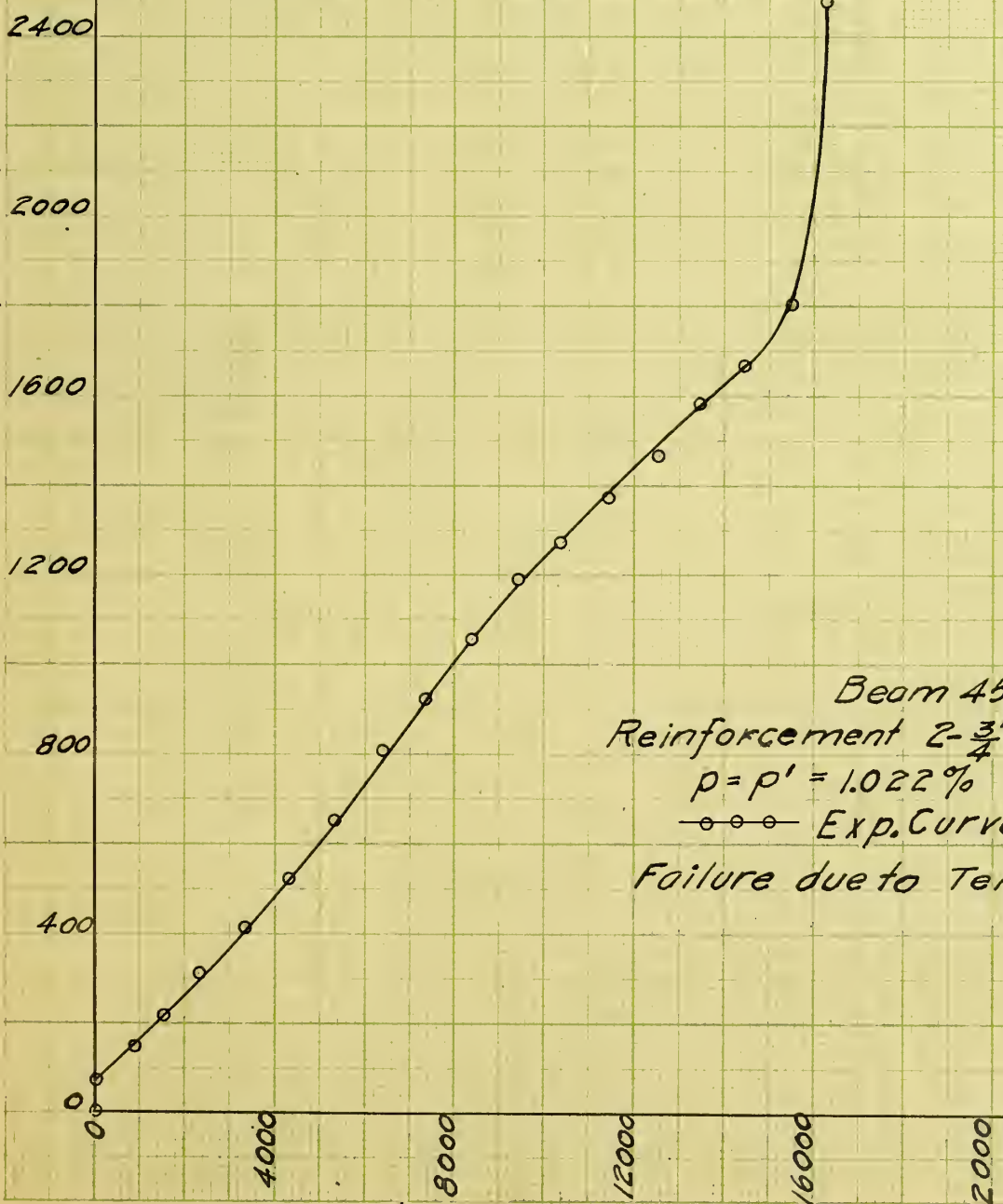
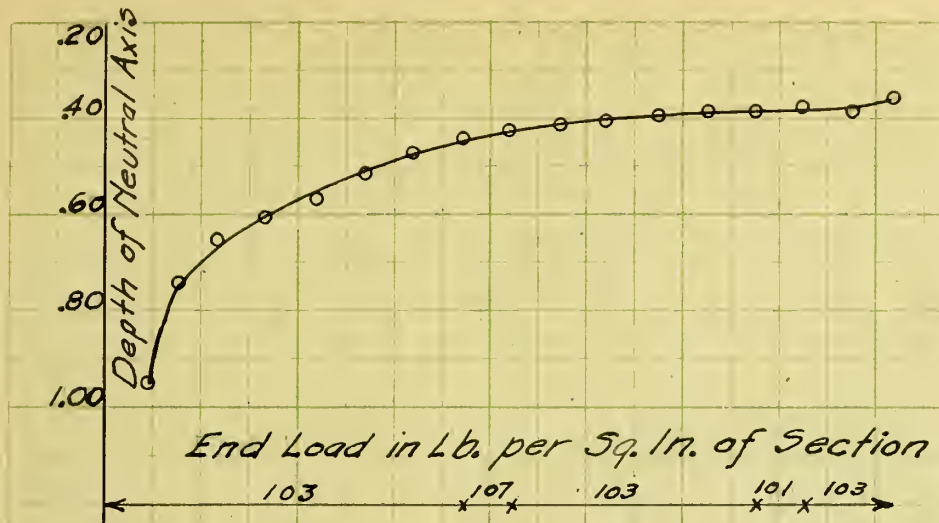
Stress in Steel Lb. per sq. in. Tension



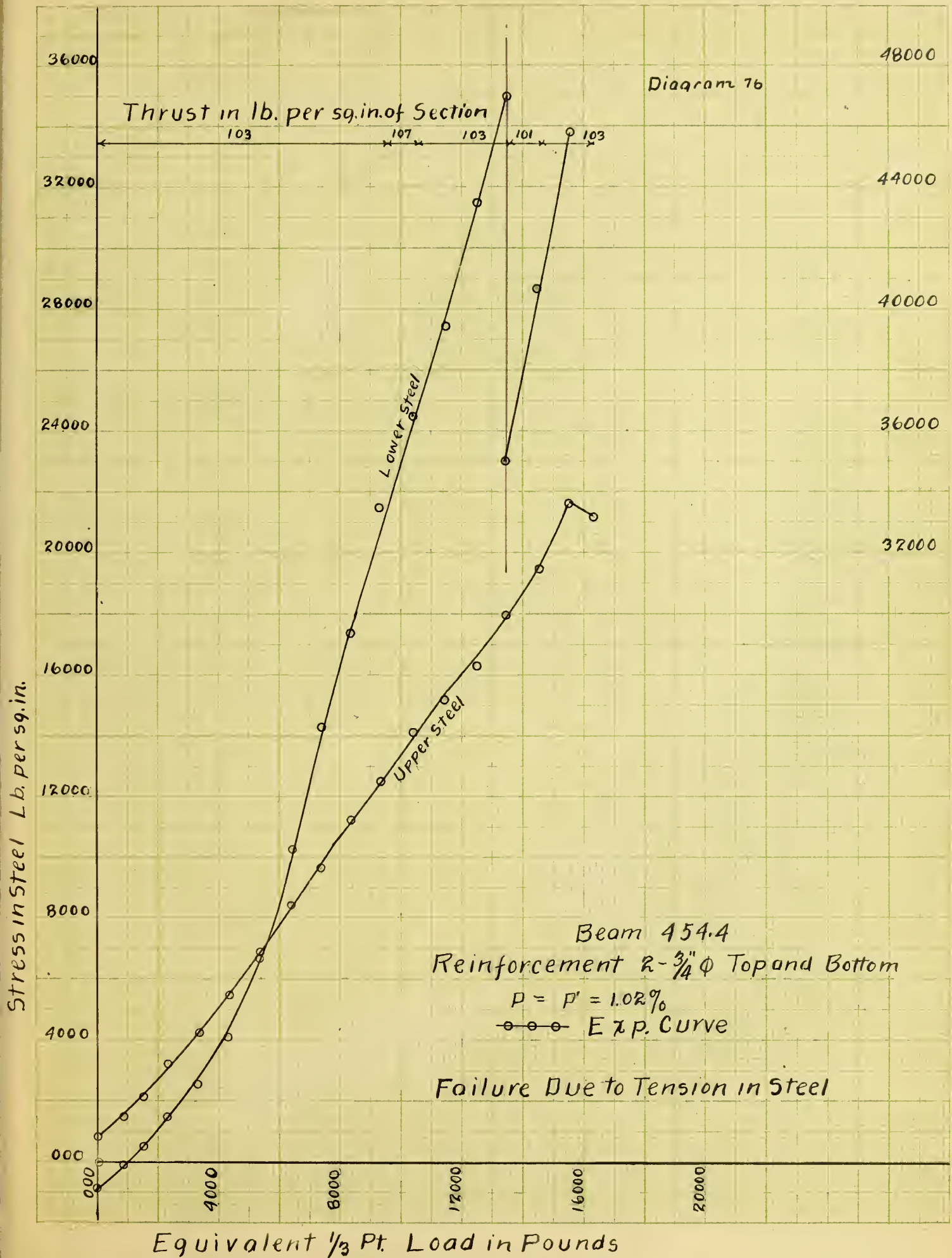
Equivalent 1/3 Pt. Load in Pounds

Diagram 75

Stress in Concrete Lb. per Sq. In.



Beam 454-4
 Reinforcement $2-\frac{3}{4}" \phi$ Top and Bottom
 $p = p' = 1.022\%$
 —○—○— Exp. Curve
 Failure due to Tension in Steel



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